CZECH TECHNICAL UNIVERSITY IN PRAGUE
FACULTY OF INFORMATION TECHNOLOGY

ASSIGNMENT OF MASTER'S THESIS

Title:	Evaluation of Data from The Viewpoint of Chaos Theory
Student:	Bc. Miroslav Škoula
Supervisor:	prof. Dr. Ing. Petr Kroha, CSc.
Study Programme:	Informatics
Study Branch:	Web and Software Engineering
Department:	Department of Software Engineering
Validity:	Until the end of winter semester 2017/18

Instructions

Time series that describe behavior of markets contain a mixture of trends and chaotic segments. The goal of this thesis is to find whether a new indicator that is based on variables describing a measure of chaos (e.g., Hurst exponent) can be included into the technical analysis, and how much profit it can bring.
1. Get to know the problem and related papers.
2. Write a prototype in Matlab that analyses chaos properties of time series and uses strategies for generating BUY- and SELL-signals.
3. Use methods of hypothesis testing to decide whether the generated BUY- and SELL-signals bring more profit than standard indicators, e.g., MACD.
4. Evaluate and summarise the achieved results.
This thesis is research oriented.

References

Peters, E.E.: Fractal Market Analysis. Wiley, 1994.
Gneiting, T., Ševčíková, H., Percival, D.B.: Estimators of Fractal Dimension: Assessing the Roughness of Time Series and Spatial Data. Statistical Science 2012, Vol. 27, No. 2, 247–277, 2012.
Sánchez Granero, M.A., Trinidad Segovia, J.E., García Pérez, J.: Some comments on Hurst exponent and the long memory processes on capital markets. Physica A 387 , pp. 5543–5551, 2008.

Ing. Michal Valenta, Ph.D.
Head of Department

prof. Ing. Pavel Tvrdík, CSc.
Dean

Prague January 3, 2015

Czech Technical University in Prague
Faculty of Information Technology
Department of Software Engineering

Master's thesis

Evaluation of Data from The Viewpoint of Chaos Theory

Bc. Miroslav Škoula

Supervisor: prof. Dr. Ing. Petr Kroha, CSc.

9th January 2018

Acknowledgements

Many thanks to all my girls for their love and support and to prof. Dr. Ing. Petr Kroha, CSc. for his guidance and patience.

Declaration

I hereby declare that the presented thesis is my own work and that I have cited all sources of information in accordance with the Guideline for adhering to ethical principles when elaborating an academic final thesis.

I acknowledge that my thesis is subject to the rights and obligations stipulated by the Act No. 121/2000 Coll., the Copyright Act, as amended. In accordance with Article 46(6) of the Act, I hereby grant a nonexclusive authorization (license) to utilize this thesis, including any and all computer programs incorporated therein or attached thereto and all corresponding documentation (hereinafter collectively referred to as the "Work"), to any and all persons that wish to utilize the Work. Such persons are entitled to use the Work in any way (including for-profit purposes) that does not detract from its value. This authorization is not limited in terms of time, location and quantity. However, all persons that makes use of the above license shall be obliged to grant a license at least in the same scope as defined above with respect to each and every work that is created (wholly or in part) based on the Work, by modifying the Work, by combining the Work with another work, by including the Work in a collection of works or by adapting the Work (including translation), and at the same time make available the source code of such work at least in a way and scope that are comparable to the way and scope in which the source code of the Work is made available.

In V Praze on 9th January 2018 ....

Citation of this thesis

Škoula, Miroslav. *Evaluation of Data from The Viewpoint of Chaos Theory.* Master's thesis. Czech Technical University in Prague, Faculty of Information Technology, 2018.

Abstrakt

Časové řady popisující chování trhů obsahují směs trendů a chaotických úseků. Cílem této práce je ukázat, zda nový indikátor, postavený na ukazatelích míry chaosu (např. Hurst exponent), je použitelný při technické analíze a jaká je jeho profitabilita v porovnání s ostatními parametry.

Klíčová slova časové řady, obchodní signály, fraktální dimenze, Hurstův exponent

Abstract

Time series that describe behavior of markets contain a mixture of trends and chaotic segments. The goal of this thesis is to find whether a new indicator that is based on variables describing a measure of chaos (e.g., Hurst exponent) can be included into the technical analysis, and how much profit it can bring.

Keywords time series, trading signals, fractal dimension, Hurst exponent

Contents

List of Figures

List of Tables

Chapter 1

Introduction

Chaos theory was developed to better describe the real world around us. Present tools like Euclidean geometry are trying to fit our infinitely complex world into few perfect and symmetrical forms. Which is good enough for a rough approximation.

The Christmas tree really look like a triangle from a distance. Like on the children's drawings. But as we come closer, we discover trunk and network of branches. All qualitatively similar to the overall shape of tree, but each different. Brunches on brunches, gradually smaller with every new generation. Size vary within each generation and every tree is different. Jet, you can recognize the Christmas tree when you see it. Euclidean geometry can't replicate the Christmas tree. It can create approximation, that will always look artificial. It can capture perceived symmetry of thee tree but not the underlying variety, the actual structure of the tree.

This self-similarity quality (branches look one like another and they all look quite like a tree itself, but still they are all different) is the defining characteristics of fractal. Most natural structures, particularly living thing, have this characteristics.

We can further extended this idea into our perception of time. Traditionally all events in time are considered either random and unpredictable or deterministic and therefor perfectly predictable. In fractal time, randomness and determinism can live together. Same, like within the tree with its outer symmetrical shape and inner chaotic network of branches.

Newton saw, as the story says, falling apple and formulated the calculus and the law of gravity. Darwin decided to go on journey and came up with theory of evolution because of it. Coincidence, we could say. But, Leibniz formulated the calculus independent on Newton in almost same time. Wallace developed the theory of natural selection without knowing of Darwin's work. These discoveries were meant to happen. We can see it as a global determinism and the way it happened as the local randomness. Again, like our tree.

This was supported by scientist who slowly left the idea of universe running like a clockwork. First blow to this deterministic view of universe was thermodynamics, which put an arrow on the time. In Newton's formulas time is reversible. Second blow was quantum mechanics and it's description of molecular structure only by state of probability. It has become apparent, that global determinism and local randomness are characterization of most natural systems and these two contrary states has to coexist. Finally, the third blow to Newtonian determinism is theory of chaos with fractals where this is possible. Determinism gives us natural laws and randomness innovation and variety.

This new thinking also required new mathematics. Compared to Euclidean geometry, where everything is simple, elegant and provable, fractals are explored by numerical experiments. We generate solution using computer and explore it's implications. This new approach is not yet fully accepted by all mathematician. It seems counterintuitive and imprecise.

The following example of mathematical experiment was used in Chaos and Order in the Capital markets by Peters [69]. Originally devised by Barnesly (1988) [5]. Firs draw a triangle and mark each top of the triangle by numbers (1,2), (3,4) and (5,6). Than pick a random point inside or outside of the triangle and label it P. Roll a fair die and draw a point halfway from P to the top of triangle labeled with the number you rolled. than repeat the same from end of this line. Repeat this 10000 times, throw out first 50 lines and you'll get Sierpinski triangle (see figure 1.1). It is infinite number of triangles encapsulated in large triangle. This self similarity is an important, although not only one, characteristics of fractals.

Chaos game will always produce Sierpinski triangle as a result, despite the fact that two random factors are present. First is the choice of starting point and the second is the dice roll. It's impossible to predict actual sequence of points. Yet, all points within the large triangle has different chance of being drawn. The empty spaces has zero chance and points outlining each triangle has greater chance. Local randomness does not mean equal probability of all possible solutions and also it does not mean independence. Each point is fully dependent on it's predecessor. Fractal statistics will therefore be different from Gaussian.

We mention Fractal a lot. But what is it? There is no mathematical definition of the term Fractal until today. We know it when we see it, but we have a hard time describing it. Not even Benoit Mandelbrot, father of fractal geometry, developed precise formula.

Fractals have some characteristics and properties that can be measured. Most significant is self-similarity, parts are somehow related to whole. This self-similarity can be precise, like triangles in Sierpinski triangle (see figure 1.1) or qualitative. Qualitative similarity is when object or process is similar in different scale, spatial or temporal, statistically. The precise from of similarity exist only mathematically. Self-similarity makes Fractal scale-invariant. Another significant characteristics is Fractal dimension.

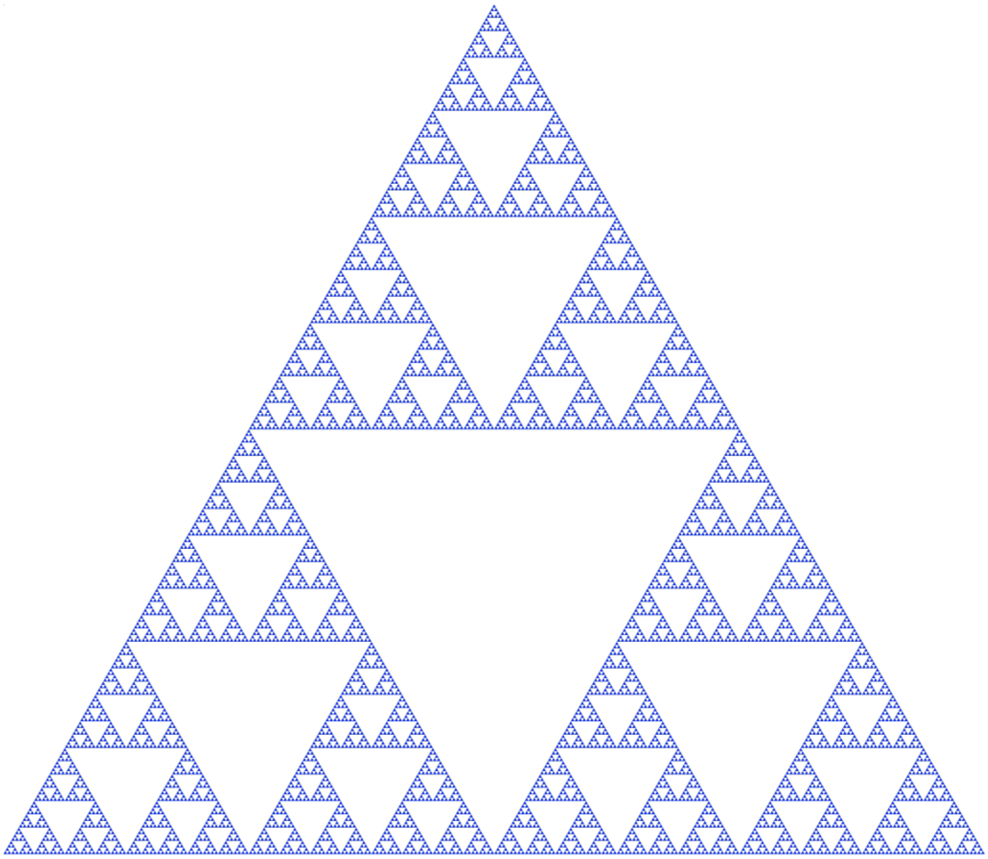

Figure 1.1: Sierpinski triangle [16]

Followings are some definitions found on the Internet.

1. **https://dictionary.cambridge.org/dictionary/english/fractal**
   noun
   a complicated pattern in mathematics built from simple repeated shapes that are reduced in size every time they are repeated.

2. **http://www.dictionary.com/browse/fractal**
   noun, Mathematics, Physics.
   a geometrical or physical structure having an irregular or fragmented shape at all scales of measurement between a greatest and smallest scale such that certain mathematical or physical properties of the structure, as the perimeter of a curve or the flow rate in a porous medium, behave as if the dimensions of the structure (fractal dimensions) are greater than the spatial dimensions.

3. **http://fractalfoundation.org/resources/what-are-fractals**
   A fractal is a never-ending pattern. Fractals are infinitely complex patterns that are self-similar across different scales. They are created by repeating a simple process over and over in an ongoing feedback loop. Driven by recursion, fractals are images of dynamic systems – the pictures of Chaos. Geometrically, they exist in between our familiar dimensions. Fractal patterns are extremely familiar, since nature is full of fractals. For instance: trees, rivers, coastlines, mountains, clouds, seashells, hurricanes, etc. Abstract fractals – such as the Mandelbrot Set – can be generated by a computer calculating a simple equation over and over.
4. **http://mathworld.wolfram.com/Fractal.html**
   A fractal is an object or quantity that displays self-similarity, in a somewhat technical sense, on all scales. The object need not exhibit exactly the same structure at all scales, but the same "type" of structures must appear on all scales. A plot of the quantity on a log-log graph versus scale then gives a straight line, whose slope is said to be the fractal dimension. The prototypical example for a fractal is the length of a coastline measured with different length rulers. The shorter the ruler, the longer the length measured, a paradox known as the coastline paradox.

Euclidean geometry has dimensions that are integers. Line is one dimensional, plane two dimensional and solids are three dimensional. Even hyperdimensions developed later are integers. For example when Einstein added time as a 4th dimension. Peters [68] used wiffle ball to show problem of this concept. Wiffle ball is hollow ball with holes in it. By definition, it's not the solid and only solids are truly three dimensional in Euclidean geometry. So, wiffel ball is less then solid but obviously, it's more thane plane. It resides in three dimensions, it has width, height and length. It's dimension is somewhere between two and three. It's non integer dimension.

Fractal dimension describes how the object fills it's space and characterize the structure at different scale. The physical fractals scale in space, time series scale statistically in time. Fractal dimension of time series measure how straight the line is. Straight line has fractal dimension of 1 and random walk has 1.5. If the fractal dimension is between 1 and 1.5, the time series is more than a line and less than a random walk. This can help us to better understand the process behind the time series. If it's more deterministic or more random. Also the statistics would be different from Gaussian for all process with fractal dimension different from 1.5.

Some methods for calculating fractal dimensions are listed in Methods chapter.

What does it all have to do with economics? Scientist have used to uses a Brownian movement to model complex processes with multiple degree of freedom. Gaussian statistics is well known and probabilities can be easily calculated. But this assumes each outcome to be independent on each other. Markets continue to be modeled like this until today. Investment and trading is equate with gambling.

Markets are assume to be efficient, that's all available information is reflected in current price of asset, to achieve the independence Gaussian statistics require. Only the random, speculative component is left out to be modeled. If the returns are not normally distributed, then we may under or over estimating our profits and risks, considering standard deviation as a measure of risk.

Figure 1.2 show that later is true and market returns are not normally distributed. Both distributions, 5 days and 90 days, have higher peak and fatter tail. It means, that the chance of large event's occurring is much higher than normal distribution implies. Greater than tree times standard deviation events occur almost five times more often. Also, the 5 days traders face the same amount of six sigma events like a 90 days investors in their investment horizon. The risk is virtually the same. Fat-tailed distribution is often evidence of long memory nonlinear stochastic process [68]. Again, we see local randomness and global determinism.

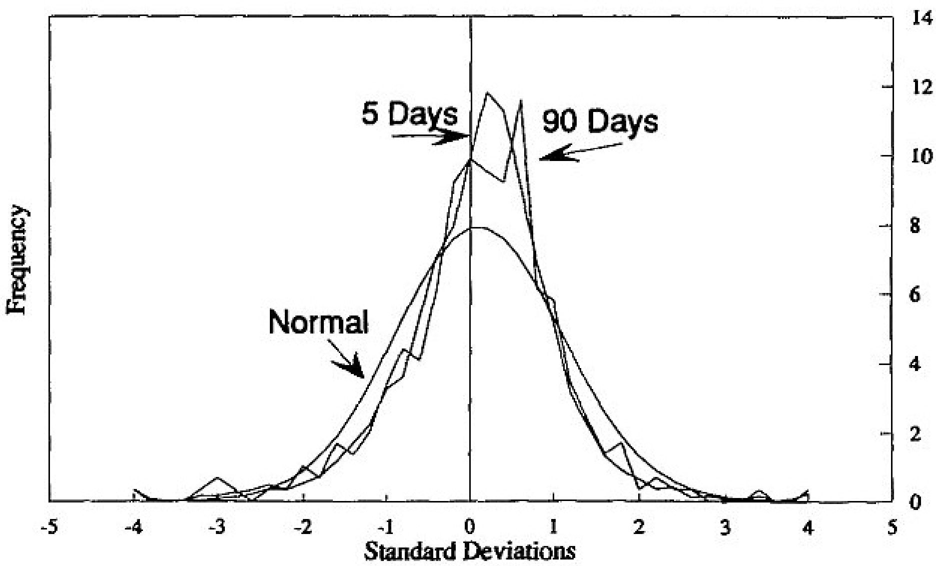

Figure 1.2: Down Jones Industrials, frequency distribution of returns 1888 - 1991 [68]

The fact, that markets don't follow the random walk and local randomness along with global determinism can be observed, led Peters [68] to formulate Fractal Market Hypothesis (FMT) as followed:

1. The market is stable when it consist of investors covering large number of investment horizons. This ensure that there is ample liquidity for trades.
2. The information set is more related to market sentiment and technical factors in short term than in long term. As investment horizon increase, longer-term fundamental information dominates.
3. If an event occurs that makes the validity of fundamental information questionable, long-term investors either stop participating in the market or begin trading based on short-term information set. When the overall investment horizon of the market shrink to uniform level, the market becomes unstable. There are no long-term investors to stabilize the market by offering liquidity to short-term investors.
4. Prices reflect a combination of short-term technical trading and long-term fundamental valuation, or "noisier", than long-term trades. The underlying trend in the market is reflective of changes in expected earnings, based on the changing economics environment. Short-term trends are more likely the result of crowd behavior. There is no reason to believe that the length of short-term tends is related to the long-term economic trend.
5. If a security has no ties to the economic cycle, then there will be no longterm trend. Trading, liquidity, and short-term information will dominate.

This fractal characteristic of markets made us believe, that using tools from the chaos toolbox can have positive impact on trading strategies.

Chapter 2

Related work

The topic Fractals and Markets is covered by very interesting and famous publication. Financial data have intuitively a fractal nature. Time series of prices in 15 minutes interval have a very similar shape like time series of daily close prices.

In his book [64], Mandelbrot collected his papers on the application of the Hurst exponent to financial time series.

Both Peters's books [68], [69] explain and discuss the Hurst exponent and its calculation using the rescaled range analysis (R/S analysis). Conclusions of Peters support the idea that there is indeed local randomness and a global structure in the financial market.This is described also in [70]. Unfortunately, Peters only applies Hurst exponent estimation to a few time series and does not discuss the accuracy of Hurst exponent calculation for sets of stock prices.

In the book [63], long-memory processes in stock market prices is discussed. But the authors, Lo and MacKinlay, do not find long-term memory in stock market return data sets they examined.

The work [47] applies the Hurst exponent to the analysis of corporate profits.

Methods that compute fractal dimension or Hurst exponent are described in overview in [52]. The problem is that all of them are estimators and deliver value that differ. One improvement of the R/S analysis is proposed in [62], and it is criticized in [73].

In his book [69], Peters suggests that a Hurst exponent value H ($0.5 < H < 1.0$) shows that the efficient market hypothesis is incorrect. Returns are not randomly distributed, there is some underlying predictability.

We investigated the topic of fractal dimension in markets in our paper [60] and compared the fuzzy and fractal technology.

The books and papers cited above have been written by professionals in statistics, and their goal was to investigate statistical entities. In our paper, we do not solve problems of statistics and the predictability question.

Our goal is to investigate the impact of changes of the Hurst exponent on trading strategies. We did not find papers discussing exactly this topic.

Chapter 3

Methods

In this chapter methods needed to develop and test our new technical indicator for a market's trends are disscused. Therefore, a first section is dedicated to technical analysis, it's presumptions and difference from fundamental analysis. We touched this topic at introduction where FMH propose valuation of information based on investment horizons.

Following section's topic is Smooth. Smoothing or filtering was used to set the trading frequency and avoid unwanted "noise" to better up the output of our new indicator. Filter was one of the last component added into our mixture and it proved to be one of the key parameters to set.

At first, we worked only with daily Close price of DAX and NASDAQ stocks. This led us nowhere, so we started to use Returns for N days, where N was another key parameter to set for a right performance. What the Return is and how is it calculated is shown in section Return N days.

To test performance of our new algorithm we used concurrent strategies Buy&Hold and MACD. One section is dedicated to each one of them. Here we also mention Omniscient trader, theoretical concept of almighty investor who knows the very best time to buy and sell every time it comes. This give us an idea of maximal possible outcome from trading.

At last but not least we describe methods from chaos toolbox used in this work. We tested two implementations of Hurst index estimator.

3.1 Technical analysis

There are two primary methods used to analyze securities and make investment decisions: fundamental analysis and technical analysis. Fundamental analysis involves analyzing a company's financial statements to determine the fair value of the business, while technical analysis assumes that a security's price already reflects all publicly-available information and instead focuses on the statistical analysis of price movements [4].

There are many different forms of technical analysis: Some rely on chart patterns, others use technical indicators and oscillators, and most use a combination of techniques. In any case, technical analysts' exclusive use of historical price and volume data. Unlike fundamental analysts, technical analysts don't concern themselves with a stock's valuation – the only thing that matters are past trading data and what information the data might provide about future price movements. [3]

Technical analysis is based on three assumptions:

1. Human behavior is often erratic and driven by emotions.
   Technical analysis can be thought of as the study of collective investor psychology or sentiment. Prices in any freely traded market are set by human beings or their automated proxies (such as computerized trading programs), and price is set at the equilibrium between supply and demand at any instant in time. Various fundamental theorists have proposed that markets are efficient and rational, but technicians believe that humans are often irrational and emotional and that they tend to behave similarly in similar circumstances. [2]

2. Market tends and patterns reflect irrational human beahvior.
   Although fundamental data are key inputs in the determination of value, these data are analyzed by humans, who may be driven, at least partially, by factors other than rational factors. Human behavior is often erratic and driven by emotion in many aspects of one's life, so technicians conclude that it is unreasonable to believe that investing is the one exception where humans always behave rationally. Technicians believe that market trends and patterns reflect this irrational human behavior. Thus, technical analysis is the study of market trends or patterns. And technicians believe the trends and patterns tend to repeat themselves and are, therefore, somewhat predictable. So, technicians rely on the recognition of patterns that have occurred in the past in an attempt to project future patterns of security prices. [2]

3. Trends and patterns repeat themselves and are thus predictable.
   Market participants use many inputs and analytical tools before trading. Technical analysts believe that emotions play a role. Investors with a favorable fundamental view may nonetheless sell a financial instrument for other reasons, including pessimistic investor sentiment, margin calls, and requirements for their capital—for example, to pay for a child's college tuition. Technicians do not care why market participants are buying or selling, just that they are doing so. [2]

3.2 Smooth

For a data filtering the smooth function of Matlab was used.

yy = smooth(y) smooths the data in the column vector y using a moving average filter. Results are returned in the column vector yy. The default span for the moving average is 5.

The first few elements of yy are given by
$yy(1) = y(1)$
$yy(2) = (y(1) + y(2) + y(3))/3$
$yy(3) = (y(1) + y(2) + y(3) + y(4) + y(5))/5$
$yy(4) = (y(2) + y(3) + y(4) + y(5) + y(6))/5$
...

yy = smooth(y,span) sets the span of the moving average to span. span must be odd.

yy = smooth(y,method) smooths the data in y using the method method and the default span. Supported values for method are listed below.

1. 'moving' - Moving average (default). A lowpass filter with filter coefficients equal to the reciprocal of the span.

2. 'lowess' - Local regression using weighted linear least squares and a 1st degree polynomial model

3. 'loess' - Local regression using weighted linear least squares and a 2nd degree polynomial model

4. 'sgolay' - Savitzky-Golay filter. A generalized moving average with filter coefficients determined by an unweighted linear least-squares regression and a polynomial model of specified degree (default is 2). The method can accept nonuniform predictor data.

5. 'rlowess' - A robust version of 'lowess' that assigns lower weight to outliers in the regression. The method assigns zero weight to data outside six mean absolute deviations.

6. 'rloess' - A robust version of 'loess' that assigns lower weight to outliers in the regression. The method assigns zero weight to data outside six mean absolute deviations.

yy = smooth(y,span,method) sets the span of method to span. For the loess and lowess methods, span is a percentage of the total number of data points, less than or equal to 1. For the moving average and Savitzky-Golay methods, span must be odd (an even span is automatically reduced by 1).

yy = smooth(y,'sgolay',degree) uses the Savitzky-Golay method with polynomial degree specified by degree.

yy = smooth(y,span,'sgolay',degree) uses the number of data points specified by span in the Savitzky-Golay calculation. span must be odd and degree must be less than span.

yy = smooth(x,y,...) additionally specifies x data. If x is not provided, methods that require x data assume x = 1:length(y). You should specify x data when it is not uniformly spaced or sorted. If x is not uniform and you do not specify method, lowess is used. If the smoothing method requires x to be sorted, the sorting occurs automatically (see figure 3.1)[15].

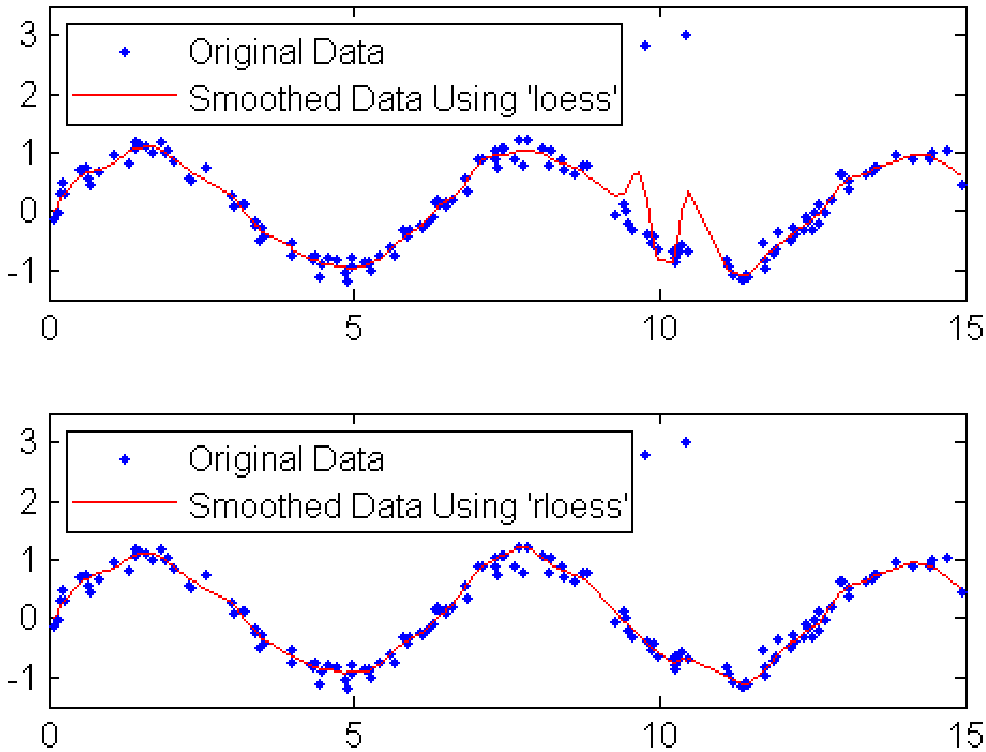

Figure 3.1: Smooth function [15]

3.3 Return N days

Return is profit on an investment. It can be absolute, measured in Dolars for example or relative, described as a percentage of investment portfolio.

Most financial models do not attempt to model close prices, but instead deal with returns on the instrument (a stock, for example). This is heavily reflected in the literature of finance and economics [13]. The return is the profit or loss in buying a share of stock, holding it for some period of time and then selling it. The most common way to calculate a return is the log return. This is shown in equation 3.1, which calculates the log return for a share of stock that is purchased and held delta days and then sold. Here $price(i+n)$ and $price(i)$ are the prices at time $i+n$ (when we sell the stock) and i (when we purchased the stock, n days ago).

Equations 3.1 and 3.2 yield similar results. We used later, although finance literature use mostly the log return.

$$return(i+n) = log(price(i+n)) - log(price(i)) \tag{3.1}$$

$$return(i+n) = \frac{price(i+n) - price(i)}{price(i)} \tag{3.2}$$

3.4 Buy&Hold

Buy and hold is a passive investment strategy in which an investor buys stocks and holds them for a long period of time, regardless of fluctuations in the market. An investor who employs a buy-and-hold strategy actively selects stocks using fundamental analysis and evaluating it's intrinsic value, by examining related economic, financial and other qualitative and quantitative factors. But once in a position, is not concerned with short-term price movements and technical indicators [27]. Buy&Hold strategy don't generate additional cost in trading fees like other trading strategies that employ more active approach with higher portion of buys and sells.

3.5 MACD

Moving Average Convergence / Divergence (MACD) is one of the basic technical trend indicators. As it's name suggests, it follows moving averages, it's convergence and divergence. Indicator MACD was originally developed by Gerald Appel at 1960. He used tree exponential moving averages (EMA). Difference between first two EMAs with length 12 and 26 makes MACD.

$$MACD = EMA(12) - EMA(26) \tag{3.3}$$

To indicate buy and sell signals, third EMA with length 9 is used. This makes signal line.

$$Signal = EMA(9) \tag{3.4}$$

Readability of indicator was improved by Thomas Aspray at 1986 who showed MACD as a histogram.

$$Histogram = MACD - Signal \tag{3.5}$$

Although, the original settings 12-26-9 is widely used, some other configuration became popular as well, 5-34-7 is one of them. In principle, first two EMAs should follow market cycles. First EMA should be set at one quarter of dominant instruments cycle and the second one at one half. Third EMA doesn't follow the cycle at all.

Sell signal is generated when the MACD line cross the signal line from up side down. Buy signal is generated when MACD line cross the signal line from down side up (see Figure 3.2).

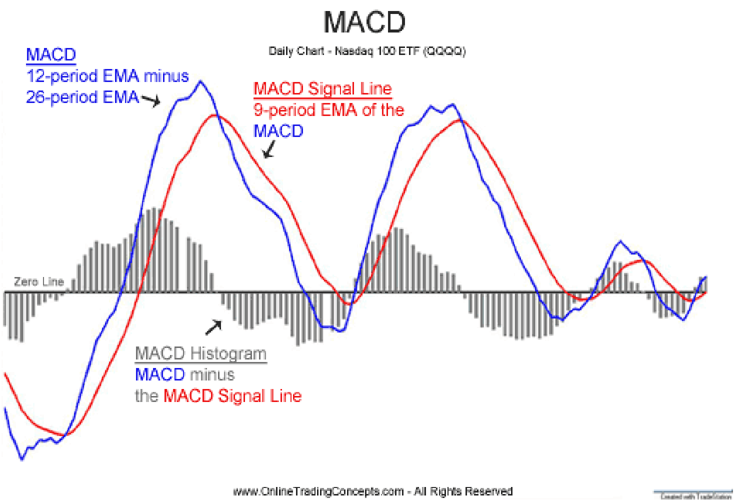

Figure 3.2: MACD [12]

3.6 Omniscient trader

Omniscient trader is theoretical concept of almighty investor who knows the very best time to buy and sell every time. Using this model, we can calculate maximum possible profit from given security. Knowing the maximum possible profit gives us clear picture of how efficient are our current trading strategies and how much room for improvement do we have. Trading activities of Omniscient trader are shown at figure 3.3 on Adidas stock.

3.7 Hurst

The Long-Term Storage Capacity of Reservoirs was the name of the paper where hydrologist H. E. Hurst (1900 - 1978) in 1951 firs published his new statistical methodology for separating random time series from fractal time series. This method was developed during his work on Nile River Dam Project.

In fractal geometry, the generalized Hurst exponent has been denoted by H or Hq in honor of both Harold Edwin Hurst and Ludwig Otto Hölder (1859–1937) by Benoît Mandelbrot (1924–2010) [30]. H is directly related to

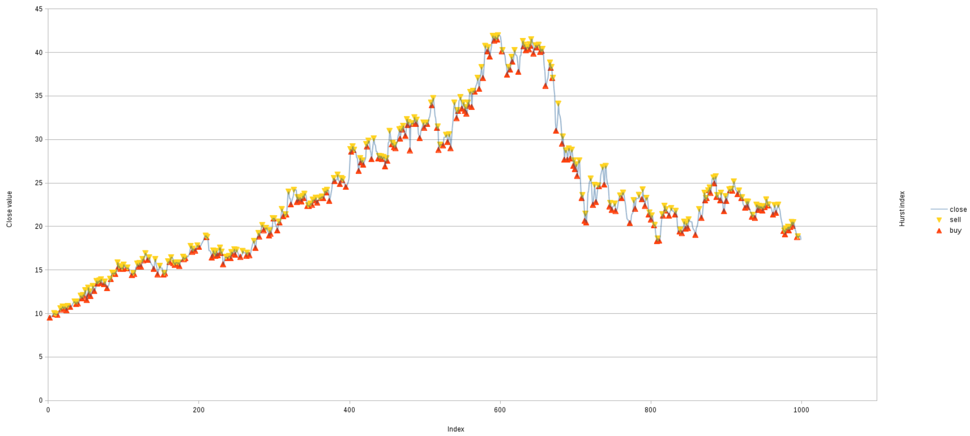

Figure 3.3: Test03 Adidas Omniscience trader, detail

fractal dimension, D, and is a measure of a data series randomness [31] This relation is shown in equation 3.6.

$$D = 2 - H \tag{3.6}$$

Theory says, that $H = 0.50$ indicate independent process. This doesn't mean just Gaussian process, but also process like Student-t, or gamma, or any other shape. R/S is nonparametric and there is no requirement for the shape of underlying distribution.

$0.50 < H \leq 1.00$ indicates persistent time series with long memory. What happens today impacts the future. This long memory occurs regardless of time scale. Daily changes are correlated with all future daily changes and weekly changes will correlate with all future weekly changes. No characteristics time scale is is the key characteristics of fractal time series [68].

$0.00 \leq H < 0.50$ indicates antipersistent system. It covers less distance than a random one.

A number of estimators have been proposed in the literature. The oldest and best-known is the rescaled range (R/S) analysis popularized by Mandelbrot and Wallis[30][36] based on Hurst findings [28]. Alternatives include DFA, Periodogram regression [37], aggregated variances [38], local Whittle's estimator [39], wavelet analysis [40][41], both in the time domain and frequency domain.

3.7.1 Definition

This definition is taken from *wikipedia* but it's in line with Peters and others. The Hurst exponent, H, is defined in terms of the asymptotic behavior of the rescaled range as a function of the time span of a time series as follows [33] [34].

$$E\left[\frac{R(n)}{S(n)}\right] = Cn^H \text{ as } n \to \infty \tag{3.7}$$

where:

- $R(n)$ is the range of the first n values, and $S(n)$ is their standard deviation
- $E\left[x\right]$ is the expected value
- n is the time span of the observation (number of data points in a time series)
- C is a constant

3.7.2 Rescaled range (R/S) analysis

To estimate the Hurst exponent, one must first estimate the dependence of the rescaled range on the time span n of observation [34]. A time series of full length N is divided into a number of shorter time series of length $n = N, N/2, N/4, \ldots$ The average rescaled range is then calculated for each value of n.

For a (partial) time series of length $n, X = X_1, X_2, \ldots, X_n$, the rescaled range is calculated as follows [33][34]:

$$m = \sum_{i=1}^{n} X_i \tag{3.8}$$

$$Y_t = X_t - m \text{ , for } t = 1, 2, 3, \ldots, n \tag{3.9}$$

$$Z_t = \sum_{i=1}^{t} Y_i \text{ , for } t = 1, 2, 3, \ldots, n \tag{3.10}$$

$$R(n) = max(Z_1, Z_2, Z_3, \ldots, Z_n) - min(Z_1, Z_2, Z_3, \ldots, Z_n) \tag{3.11}$$

$$S(n) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i - m)^2} \tag{3.12}$$

The Hurst exponent is estimated by fitting the power law 3.7 to the data. This can be done by plotting

$$log\left[\frac{R(n)}{S(n)}\right] \tag{3.13}$$

as a function of $log(n)$, and fitting a straight line. The slope of the line gives H (a more principled approach fits the power law in a maximum-likelihood fashion [42]). Such a graph is called a pox plot. However, this approach is known to produce biased estimates of the power-law exponent. For small n there is a significant deviation from the 0.5 slope. Anis and Lloyd [43] estimated the theoretical (i.e., for white noise) values of the R/S statistic to be [1]:

$$E\left[\frac{R(n)}{S(n)}\right] = \begin{cases} \frac{\Gamma\left(\frac{n-1}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{n}{2}\right)} \sum_{i=1}^{n-1}\sqrt{\frac{n-i}{i}} & \text{for } n \leq 340 \\ \frac{1}{\sqrt{n\frac{\pi}{2}\left(\frac{n}{2}\right)}} \sum_{i=1}^{n-1}\sqrt{\frac{n-i}{i}} & \text{for } n > 340 \end{cases} \tag{3.14}$$

3.7.3 Accuracy of Hurst Exponent Estimators

We had in hand two implementation of Hurst's rescale analysis algorithms. One, written by Rafal Weron 2011 and the other one written by Ian Kaplan 2003. We needed to select one as core function for our indicator and select the right configuration for it. To do that, we tested both functions of datasets with known Hurst exponent. Data sets are listed below.

1. brown72 with $H = 0.72$ [69].

2. fgn8 with $H = 0.80$

Rafal Weron's algorithm have a box size d as a parameter and returns uncorrelated Hal, empirical He and theoretical Hurst exponent Ht. It also returns confidence interval 95% $pval95$. Question was, which one of output exponents should we use, Hal, He or Ht? What box size d should we set? And how little data do we need to get still believable result. Table 3.1 show the results of Weron's algorithm on dataset brown72 ($H = 0.72$) for a different box size d. Table 3.3 show the same for a second data set, fgn8 ($H = 0.80$).

From the results we fixed $d = 8$ as best settings and tryed algorithm with different data size $n = 1024, 512, 256, \ldots, 32$ table 3.5 show the results on dataset brown72 and the table 3.7 on dataset fgn8. To shrink the data set size we picked every second ($n = 512$), forth ($n = 256$), eighth ($n = 128$), sixteenth ($n = 64$) and thirty second member ($n = 32$) hoping, that by doing so we didn't change its Hurst exponent much from it's original level.

Kaplan's implementation of R/S analysis have no input parameter apart from tested data. Also, it only give one output, exponent H. So, the testing was a bit easier. We tested only different size of datasets. Table 3.9 show the results with the brown72 dataset ($H = 0.72$) and table 3.11 show the results with the fgn8 dataset ($H = 0.80$).

Weron's implementation show better results. The empirical He is right on spot with box size $d = 8$ and all available data. We can say that box size affect the result much less than size of the dataset, where undoubtedly bigger is better.

Table 3.1: Weron on Brown72 ($H = 0.72$), different d

n	**d**	**Hal**	**He**	**Ht**	**pval95**
1024	2	0.63	0.76	0.62	[0.35,0.64]
1024	4	0.64	0.74	0.60	[0.35,0.64]
1024	8	0.66	0.73	0.57	[0.35,0.64]
1024	16	0.66	0.72	0.56	[0.35,0.64]
1024	32	0.67	0.71	0.54	[0.35,0.64]
1024	64	0.67	0.70	0.54	[0.35,0.64]
1024	128	0.77	0.79	0.53	[0.35,0.64]
1024	254	0.83	0.85	0.52	[0.35,0.64]

Table 3.2: *

n - sample size, d - box size, Hal - uncorrelated, He- empirical and Ht - theoretical Hurst exponent, $pval95$ confidence interval 95%

Table 3.3: Weron on fgn8 ($H = 0.80$), different d

n	**d**	**Hal**	**He**	**Ht**	**pval95**
1024	2	0.68	0.81	0.62	[0.35,0.64]
1024	4	0.70	0.80	0.60	[0.35,0.64]
1024	8	0.72	0.79	0.57	[0.35,0.64]
1024	16	0.73	0.78	0.56	[0.35,0.64]
1024	32	0.74	0.78	0.54	[0.35,0.64]
1024	64	0.73	0.76	0.54	[0.35,0.64]
1024	128	0.54	0.56	0.53	[0.35,0.64]
1024	254	0.48	0.49	0.52	[0.35,0.64]

Table 3.4: *

n - sample size, d - box size, Hal - uncorrelated, He- empirical and Ht - theoretical Hurst exponent, $pval95$ confidence interval 95%

Table 3.5: Weron on Brown72 ($H = 0.72$), different n

n	**d**	**Hal**	**He**	**Ht**	**pval95**
1024	8	0.66	0.73	0.57	[0.35,0.64]
512	8	0.60	0.68	0.58	[0.29,0.70]
256	8	0.53	0.63	0.60	[0.19,0.79]
128	8	0.41	0.50	0.61	[-0.02,0.98]
64	8	0.48	0.60	0.64	[-0.44,1.35]
32	8	0.40	0.54	0.67	[-1.56,2.35]

Table 3.6: *

n - sample size, d - box size, Hal - uncorrelated, He- empirical and Ht - theoretical Hurst exponent, $pval95$ confidence interval 95%

Table 3.7: Weron on fgn8 ($H = 0.80$), different n

n	**d**	**Hal**	**He**	**Ht**	**pval95**
1024	8	0.72	0.79	0.57	[0.35,0.64]
512	8	0.67	0.75	0.58	[0.29,0.70]
256	8	0.62	0.73	0.60	[0.19,0.79]
128	8	0.63	0.75	0.61	[-0.02,0.98]
64	8	0.63	0.79	0.64	[-0.44,1.35]
32	8	0.75	0.99	0.67	[-1.56,2.35]

Table 3.8: *

n - sample size, d - box size, Hal - uncorrelated, He- empirical and Ht - theoretical Hurst exponent, $pval95$ confidence interval 95%

Table 3.9: Kaplan on Brown72 ($H = 0.72$), different n

n	**H**	**slopeError**
1024	0.73	0.011
512	0.70	0.006
256	0.60	0.022
128	0.49	0.046
64	0.43	0.082
32	0.29	0.114

Table 3.10: *

n - sample size, H - Hurst exponent, $slopeError$

Table 3.11: Kaplan on fgn8 ($H = 0.80$), different n

n	**H**	**slopeError**
1024	0.75	0.019
512	0.70	0.029
256	0.61	0.062
128	0.69	0.026
64	0.63	0.072
32	0.70	0.137

Table 3.12: *

n - sample size, H - Hurst exponent, $slopeError$

Chapter 4

Implementation

Prototype of the application that analyses the time series using Hurst exponent and generating BUY- and SELL-signals was developed in Matlab.

Goal was to develop system where independent test can be created, run and re-run if necessary and the results are stored into csv files for further evaluation. Although tests are independent, they compose of the same components. These components are packed into functions that are shared among all tests.

Main unit is a testN script that manage loading data and store aggregated test results. Test itself is manager by function profitUsingReturnsAndHurstN that applies different strategies and stores each individual result. This two are unique all other features are wrapped into functions and shared among profitUsingReturnsAndHurstN functions. Usually one profitUsingReturnsAndHurstN function is shared between two tests, one for DAX and the other for NASDAQ stocks.

Some of the functions were implemented for this work (testN, profitUsingReturnsAndHurstN, buyAndHold, returnNdays, movingHurst), others are Matlab toolbox or taken over from previous work on similar topic. Some function had to be reimplemented from other language like myHurst function by Ian Kaplan 2003 was reimplemented from C++. Hurst function by Rafal Weron 2011 had to be fixed for better stability. It crashed for datasets that has two or more same local minimums or maximums in them. Function struct2csv was modified to write array of structures into csv file, previously it writes only single structure into csv file.

From two available R/S analysis implementations we choose the Ian Kaplan's function mainly for it's better performance, thought lower computation time. Weron's implementation, on the other side, far more accurate. Indicator was designed to monitor differences in Hurst exponent on different data windows. Accuracy was therefor not a problem when the results and the error was stable. Performance was a problem, we tested lots of configuration before getting somewhere. Over all, tests generated over 4 millions results. Testing of one configuration often took more than couple of hours on personal computer.

4.1 Requirements

Requirements for the program where few:

1. Program Hurst based indicator for BUYs and SELLs.
2. Test it with other conventional methods.
3. Allow to re-run all the test.
4. Collect data for a later evaluation.
5. Non-functional, make it possible on personal computer in reasonable time.

Only use case for this application is to run tests and store results into csv files figure 4.1.

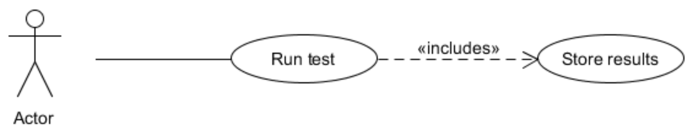

Figure 4.1: Use case

4.2 Structure

To describe structure of function based program we couldn't find proper notation. That's why we modified UML's class diagram notation into our function dependency diagram. At top of the box is the name of function. Below the line are functions parameters and at the bottom are responsibilities of each function. Multiplicity along the call lines show the reader following:

- 0 at calling function side indicate that called function is independent on calling function. In Matlab it can mean that the function is declared separately in its on file and can be used without calling function.
- n at calling function side indicate that calling function exist in more than one version, notice N (version number placeholder) in the name of those functions in figure 4.2

- $0..x$ (where x is $1, 2 \ldots n$) at called side indicates that the call is not mandatory but optional depending on configuration, algorithms evaluation path or for other reasons.

- 1 at called side indicates that the call is made one, with one configuration, one set of prarameters

- n (where $n > 1$) at called side indicates that the call is made more than once with different configuration, different version of same function and so on.

Arrows at call lines directs from calling to called function, if this can be mistaken.

Let focus now on the structure of presented program. Main unit is the Test$N$. Each test has it's own original file (version) Test 03 - Test 33. This main function loads the test time series, DAX or NASDAQ stock using importIndexFile or importStockFile functions. Runs the test on each stock using profitUsingReturnAndHurstN function and stores the resultusing structToCsv function to write MATLAB structure into *.csv file. ProfitUsingReturnAndHurstN functions is the core function of the program. Here, the main logic of the indicatr (each version is written). It trades on given stock data and compares the result with other known methods, MACD, Buy & Hold and Omniscient trader. Important function is the movingHurst. It can calculate Hurst exponent time series where each point is Hurst exponent for a past n days window, where n is *sampleSize* parameter of the function.

All the functions, it's responsibilities and dependencies are shown in function dependency diagram in figure 4.2.

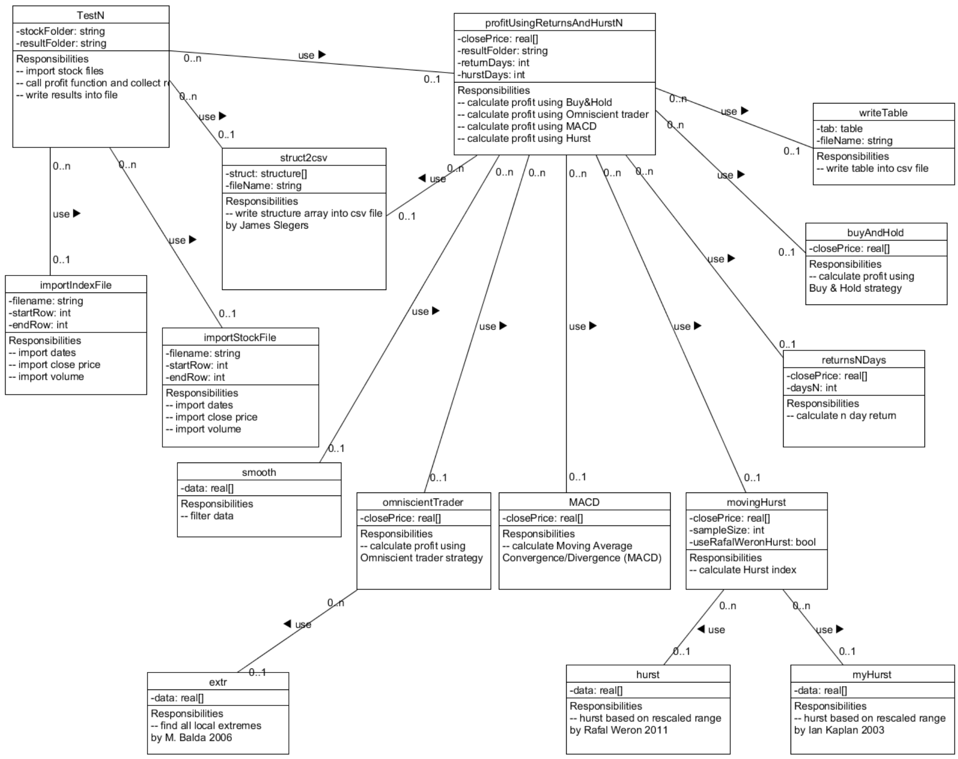

Figure 4.2: Function dependency diagram

CHAPTER 5

Results

The goal of this work was to develop a new technical indicator that is based on Hurst exponent evaluating method and compare it with other existing and already used technical indicators like MACD and investment strategy Buy & Hold. Additionally, the theoretical indicator with the best possible profit, Omniscient trader was used for a compaction. All methods are described in chapter Methods 3.

The idea was to start with statical levels for a generating BUY and SELL signals. Once the Hurst exponent signal line cross the BUY level from bottom up, the BUY signal is generated. Once the Hurst exponent signal line cross the SELL level from up down, the SELL signal is generated. First task was to prove that such a BUY and SELL level exists with knowledge of all data in section 5.1.1.

When we learned that such levels can be found. We tried to develop methods that would use past trading data to find the levels for BUY and SELL and then apply them on new data, generating profit. This cen be found in section 5.1.2. We tried different length of teach and test periods from half by half cut to 100 days piece for a teaching and another 100 days piece for a testing. We coudn't find any combination that would outperformed the traditional Buy & Hold strategy.

Next step was to introduce some kind of dynamical system that would use recent history to indicate BUY and SELL points. We used MACD mechanism and use one fast Hurst exponent line and one slow Hurst exponent line that cross each other and generate BUY and SELL signal at each crossing. This prove to be a successful approach and two winning test one for a DAX and one for NASDAQ are presented in section 5.1.3.

Detailed description of all tests can be found in appendix D along with the list of all the data and time period. DAX stock is listed here C.0.1 and NASDAQ here C.0.2.

5.1 Experiments

5.1.1 Prove of concept

First working hypothesis was: Such levels BUY and SELL can be found, that when crossed by Moving Hurst line, signals BUY and SELL are generated. Following those signal in trade would create more profit than conventional investment strategies.

Before hunting for a self teaching algorithm, we wanted to know if this is possible to achieve with knowledge of all date. Brute force method was introduced and such levels found, see picture figure 5.1 BUY signal is generated when signal line cross BUY level from bottom up. SELL signal is generated when signal line cross SELL level from up down.

Goal was to set BUY and SELL level to maximize the profit and top-up MACD and Buy & Hold strategy. All history was used for a training. Levels found by the test generated more profit then other competing strategies MACD and Buy & Hold. Omniscient trader will always be better then any other strategy from principle of this indicator.

- test03 DAX table 5.1 table 5.3

  average profit Omniscient trader: 1629.8

  average profit Buy& Hold: 41.6

  average profit MACD: 25.8

  average profit Hurst: 81.6

- test04 NASDAQ table 5.7

  average profit Omniscient trader: 1015.6

  average profit Buy& Hold: 45.1

  average profit MACD: 7.5

  average profit Hurst: 72.7

5.1.2 Teach and test

The idea was to use part of the data for a training and then use level BUY and level SELL on the rest of the data to generate profit using same BUY and SELL signals like in prove of concept test. We tried to split data in half and use firs half for a training and the other half for a testing. Then we tried smaller and smaller pieces up to 100 days long piece for a training and 100 days piece for a trading. Unfortunately, no combination of teach and test period that would be more successful than Buy & Hold strategy was found .

Table 5.1: Test 03 DAX

Name	OT	B&H	Hurst	T	Gain	Lost	Tg	Tl	Lt	Lv
Adidas-A1EWWW.txt	1035.9	73.8	98.4	32	135.9	-37.6	22	10	0.8	0.7
Allianz-840400.txt	5251.1	-4.2	31.3	46	501.7	-470.4	25	21	0.79	0.74
BASF-BASF11.txt	956.9	59.4	62.5	2	62.5	0.0	2	0	0.6	0.54
BMW-519000.txt	1302.1	62.4	71.0	3	84.2	-13.2	2	1	0.56	0.52
Bayer-BAY001.txt	1270.9	69.6	109.8	77	180.6	-70.9	50	27	0.78	0.77
Beiersdorf-520000.txt	1021.2	56.0	69.8	319	120.0	-50.3	54	38	0.5	0.75
Commerzbank-0BK100.txt	4164.8	-102.5	180.3	42	577.4	-397.1	21	21	0.79	0.73
Continental-543900.txt	1611.1	110.3	146.5	105	242.6	-96.1	27	15	0.5	0.68
Daimler-710000.txt	1645.8	24.9	34.3	3	77.1	-42.8	2	1	0.74	0.51
Dt-Bank-514000.txt	1922.5	2.3	22.1	3	71.5	-49.4	2	1	0.68	0.51
Dt-Boerse-581005.txt	1176.0	34.1	83.9	34	169.8	-85.9	25	9	0.79	0.73
Dt-Post-555200.txt	326.2	0.7	33.5	351	63.0	-29.5	46	28	0.5	0.8
Dt-Telekom-555750.txt	708.6	-7.5	10.7	63	98.4	-87.7	29	34	0.8	0.78
EON-ENAG99.txt	621.5	2.8	26.6	80	76.3	-49.6	40	39	0.8	0.77
FMC-578580.txt	820.5	33.6	30.1	6	49.7	-19.6	3	3	0.77	0.6
Fresenius-578563.txt	1372.5	110.4	114.3	5	118.5	-4.2	4	1	0.64	0.59
Heidelberger-604700.txt	1787.1	15.0	136.4	32	208.1	-71.7	18	14	0.8	0.69
Henkel-604843.txt	838.0	67.1	69.5	33	83.5	-14.0	25	8	0.73	0.69
KaS-0SAG88.txt	798.8	15.9	53.1	61	119.3	-66.3	29	32	0.8	0.76
Linde-648300.txt	1918.1	108.3	146.0	752	262.0	-116.0	69	49	0.5	0.8
Lufthansa-823212.txt	542.3	3.3	7.6	3	14.9	-7.3	2	1	0.74	0.51
Merck-659990.txt	1673.0	88.0	102.3	54	193.2	-90.9	35	18	0.8	0.74
Muenchener-843002.txt	4898.4	65.8	212.7	49	511.8	-299.1	33	15	0.76	0.73
RWE-703712.txt	1339.7	-3.9	64.5	329	176.7	-112.2	41	29	0.5	0.77
SAP-716460.txt	1359.9	49.0	57.2	3	57.2	0.0	3	0	0.75	0.51
Siemens-723610.txt	2180.4	58.7	113.7	70	269.7	-156.0	39	31	0.77	0.75
Thyssen-750000.txt	784.8	5.8	29.7	689	106.8	-77.1	51	43	0.5	0.8
Volkswagen-766403.txt	2306.5	165.2	167.2	3	167.2	0.0	3	0	0.59	0.56
Average	**1629.8**	**41.6**	**81.6**	**116**	**171.4**	**-89.8**	**25**	**17**	**0.69**	**0.68**

Table 5.2: *

name - stock name, OT - profit of Omniscient trader, $B\&H$ - profit of Buy & Hold, Hurst - profit of tested strategy, Gains - Total gain, Lost - Total lost, Tg - no. of gain transactions, Tl - no. of lost transactions, Lt - level BUY, Lv - Level SELL

Table 5.3: Test 03 DAX index

Name	OT	B&H	Hurst	T	Gain	Lost	Tg	Tl	Lt	Lv
DAX-846900.txt	130514.707	5647.8799	7750.2299	21	13938.29	-6188.0601	16	5	0.77	0.67

Table 5.4: *

name - stock name, OT - profit of Omniscient trader, $B\&H$ - profit of Buy & Hold, Hurst - profit of tested strategy, Gains - Total gain, Lost - Total lost, Tg - no. of gain transactions, Tl - no. of lost transactions, Lt - level BUY, Lv - Level SELL

Table 5.5: Test04 NASDAQ part 1

Name	OT	B&H	Hurst	T	Gain	Lost	Tg	Tl	Lt	Lv
Avago-A0X9TN.TXT	239.8	30.6	40.2	24	47.1	-6.8	18	5	0.78	0.76
Baidu-A0F5DE.TXT	1197.7	101.0	153.9	19	194.9	-40.9	11	7	0.68	0.69
Bed-Bath-884304.TXT	1104.2	39.7	49.5	33	90.1	-40.6	18	15	0.79	0.7
Biogen-789617.TXT	2336.3	201.5	256.3	74	348.1	-91.8	45	26	0.78	0.77
Broadcom-913684.TXT	1856.5	12.4	64.3	535	200.2	-135.9	55	38	0.5	0.78
CA-Techn-A0JC59.TXT	1118.9	-8.9	41.6	166	93.2	-51.5	26	25	0.5	0.73
Catamaran-A1J08W.TXT	328.4	27.6	27.0	11	34.1	-7.1	5	1	0.5	0.61
Celgene-881244.TXT	1113.3	94.4	97.0	34	113.2	-16.2	7	4	0.5	0.64
Cerner-892807.TXT	388.1	36.7	38.3	7	40.7	-2.4	5	2	0.65	0.58
Charter-Comm-A0YF1T.TXT	258.4	52.1	89.9	9	94.5	-4.6	7	2	0.8	0.77
Check-Point-901638.TXT	1632.1	41.6	94.9	42	191.0	-96.1	23	19	0.73	0.7
Cisco-878841.TXT	882.7	10.5	50.5	59	101.6	-51.1	30	28	0.71	0.7
Citrix-898407.TXT	1558.6	34.6	42.9	14	80.6	-37.7	9	5	0.69	0.63
Cognizant-915272.TXT	383.3	35.0	36.9	11	40.3	-3.4	9	2	0.76	0.61
Comcast-157484.TXT	351.5	19.5	23.1	51	37.8	-14.7	25	23	0.78	0.76
Costco-888351.TXT	1388.2	67.0	72.5	21	99.8	-27.4	15	6	0.78	0.66
DirecTV-A1J1EZ.TXT	413.5	39.5	54.0	50	72.6	-18.6	30	20	0.8	0.77
Discovery-Comm-A0Q90G.TXT	409.3	49.6	51.3	6	51.3	0.0	6	0	0.73	0.61
Dish-Network-A0NBN0.TXT	915.7	11.8	59.8	431	137.9	-78.2	66	38	0.5	0.8
Doller-Tree-A0NFQC.TXT	420.0	25.9	22.1	5	32.2	-10.1	3	2	0.5	0.61
Equinix-165241.TXT	2443.8	-425.1	-109.3	7	139.7	-249.0	5	2	0.58	0.57
Expedia-A1JRLJ.TXT	482.8	34.7	35.8	19	51.9	-16.0	11	8	0.72	0.71
Expeditors-875272.TXT	716.6	22.1	33.0	28	58.4	-25.4	16	12	0.8	0.72
Express-Scripts-A1JWJL.TXT	589.6	48.4	53.8	11	58.7	-4.9	9	2	0.74	0.59
F5-922977.TXT	1610.2	55.6	96.8	75	199.1	-102.4	36	38	0.8	0.79
Facebook-A1JWVX.TXT	139.1	10.9	31.2	4	31.2	0.0	4	0	0.7	0.66
Fastenal-887891.TXT	443.9	28.4	33.0	25	47.1	-14.1	16	9	0.75	0.7
Fiserv-881793.TXT	517.4	30.9	48.9	51	72.4	-23.5	34	17	0.78	0.73
Garmin-A1C06B.TXT	842.1	30.7	33.3	6	77.7	-44.4	4	2	0.78	0.62
Gilead-885823.TXT	432.6	50.5	56.4	6	56.4	0.0	6	0	0.78	0.55
Google-A0B7FY.TXT	3299.2	354.7	368.2	35	548.8	-180.6	24	11	0.8	0.78
Henry-Schein-897961.TXT	907.5	75.2	78.7	19	96.2	-17.5	13	6	0.7	0.63
Illumina-927079.TXT	1151.5	58.4	152.9	18	187.6	-34.7	12	6	0.8	0.66
Intel-855681.TXT	955.5	8.4	37.2	72	98.1	-60.9	44	28	0.75	0.74
Intuit-886053.TXT	937.9	49.3	54.5	32	78.6	-24.1	21	11	0.7	0.69
Intuitivr-Surgical-888024.TXT	4579.3	264.8	386.5	32	587.0	-200.6	20	12	0.8	0.72
KLA-Tencor-865884.TXT	1741.0	30.3	41.3	5	86.6	-45.3	3	2	0.69	0.52
Keurig-A1XFME.TXT	685.1	69.4	73.6	4	82.1	-8.5	3	1	0.5	0.56
Kraft-Foods-A1J20U.TXT	76.0	5.5	41.3	3	41.3	0.0	3	0	0.65	0.64
Liberty-A0JMPL.TXT	169.7	2.7	6.6	101	22.0	-15.4	15	14	0.5	0.8
Liberty-Global-A1W0FL.TXT	511.3	10.9	37.3	43	86.2	-48.9	29	14	0.79	0.78
Liberty-Media-Corp-A1KBFW.TXT	198.9	8.6	13.1	3	16.8	-3.7	2	1	0.74	0.73
Linear-872629.TXT	1211.9	15.8	27.4	34	87.1	-59.6	19	15	0.69	0.68
Marriott-913070.TXT	686.4	25.1	40.0	41	80.5	-40.5	27	13	0.72	0.7
Mattel-851704.TXT	622.9	-6.3	6.3	19	37.2	-30.9	11	8	0.79	0.67
Maxim-Integrated-876158.TXT	1416.9	16.6	24.9	90	127.8	-102.9	42	43	0.75	0.74
Micron-869020.TXT	1172.9	0.5	79.3	21	132.9	-53.6	12	9	0.8	0.65
Microsoft-870747.TXT	798.8	23.0	29.3	24	39.2	-9.9	16	8	0.77	0.67
Mondelez-A1J4U0.TXT	323.9	2.1	8.6	31	22.8	-14.2	14	17	0.78	0.72
Monster-Beverage-A1JSKK.TXT	540.1	47.4	53.3	3	53.3	0.0	3	0	0.7	0.51
Mylan-868270.TXT	456.2	25.8	54.7	52	79.0	-24.3	33	19	0.79	0.75

Table 5.6: *

name - stock name, OT - profit of Omniscient trader, *B&H* - profit of Buy & Hold, Hurst - profit of tested strategy, Gains - Total gain, Lost - Total lost, Tg - no. of gain transactions, Tl - no. of lost transactions, Lt - level BUY, Lv - Level SELL

Table 5.7: Test04 NASDAQ part 2

Name	OT	B&H	Hurst	T	Gain	Lost	Tg	Tl	Lt	Lv
NETFlix-552484.TXT	2159.4	242.6	271.5	3	271.5	0.0	3	0	0.57	0.53
NVIDIA-918422.TXT	537.8	11.9	15.8	10	29.0	-13.2	8	2	0.66	0.62
NXP-Semiconductors-A1C5WJ.TXT	226.0	33.2	29.7	2	29.7	0.0	2	0	0.79	0.57
NetApp-A0NHKR.TXT	1763.5	18.8	42.4	86	219.0	-176.6	42	44	0.8	0.79
OReilly-A1H5JY.TXT	688.6	84.8	87.6	1	87.6	0.0	1	0	0.5	0.5
PACCAR-861114.TXT	872.9	32.8	33.2	67	71.7	-38.5	14	8	0.5	0.63
Paychex-868284.TXT	975.8	15.4	26.2	37	81.7	-55.5	18	19	0.78	0.73
Qualcomm-883121.TXT	1297.4	55.4	63.2	312	176.7	-113.5	51	29	0.5	0.74
Regeneron-Pharma-881535.TXT	1764.8	168.8	200.4	6	208.1	-7.7	5	1	0.78	0.6
Robinson-Worldwide-A0HGF5.TXT	682.5	25.1	46.3	31	70.9	-24.6	18	13	0.77	0.74
Ross-870053.TXT	477.3	45.3	47.7	10	56.2	-8.5	6	4	0.74	0.64
SBA-Comm-923376.TXT	861.5	53.3	77.5	69	112.3	-34.8	45	22	0.8	0.78
Sandisk-897826.TXT	1401.7	56.0	81.8	15	119.9	-38.2	10	5	0.76	0.64
Seagate-A1C08F.TXT	480.3	23.0	22.3	3	30.0	-7.7	1	2	0.79	0.58
Sigma-Aldrich-863120.TXT	766.4	53.2	75.3	133	97.6	-22.4	33	26	0.7	0.74
Staples-876951.TXT	567.2	1.5	8.2	31	34.1	-25.9	14	17	0.78	0.66
Starbucks-884437.TXT	627.2	46.8	47.2	29	77.5	-30.3	18	11	0.78	0.68
Stericycle-902518.TXT	807.3	72.7	87.9	7	92.3	-4.3	6	1	0.77	0.6
Symantec-879358.TXT	452.6	13.2	14.6	24	24.5	-10.0	9	1	0.5	0.57
Syrius-A1W8XE.TXT	619.1	-29.4	5.9	35	64.2	-58.3	23	10	0.78	0.71
Tesla-A1CX3T.TXT	676.0	127.1	133.2	2	133.2	0.0	2	0	0.5	0.57
Texas-Instruments-852654.TXT	1264.5	24.4	78.2	74	138.9	-60.8	44	30	0.8	0.77
Tractor-889826.TXT	359.1	43.4	47.1	19	52.5	-5.4	15	4	0.77	0.7
TripAdvisor-A1JRLK.TXT	261.7	38.1	54.9	6	69.9	-15.0	4	2	0.77	0.69
Twenty-First-Century-A1WZP6.TXT	293.6	10.6	8.4	3	10.1	-1.7	2	1	0.66	0.5
Verisk-A0YA2M.TXT	163.0	27.0	48.3	3	48.3	0.0	3	0	0.79	0.58
Vertex-882807.TXT	1439.1	23.2	100.9	77	214.2	-113.3	36	40	0.8	0.79
Viacom-A0HM1Q.TXT	1420.3	38.7	53.3	9	101.2	-47.9	3	6	0.8	0.57
VimpelCom-A0YE2R.TXT	81.7	-7.3	-3.0	14	3.8	-6.8	5	9	0.75	0.71
Vodafon-A1XD9Z.TXT	684.1	-32.4	-13.4	88	47.0	-60.4	45	43	0.8	0.79
Western-Digital-863060.TXT	899.1	35.4	66.1	42	127.0	-60.9	24	18	0.8	0.73
Whole-Foods-886391.TXT	537.5	28.0	34.8	339	75.1	-40.3	40	34	0.5	0.77
Wynn-Resorts-663244.TXT	1787.7	116.5	123.7	25	130.9	-7.2	7	1	0.5	0.63
Xilinx-880135.TXT	1476.2	31.7	85.5	41	127.8	-42.2	23	17	0.79	0.7
Yahoo-900103.TXT	1129.6	25.2	41.0	24	108.4	-67.4	15	9	0.75	0.66
eBay-916529.TXT	913.5	37.9	27.7	13	39.9	-12.2	5	4	0.5	0.62
priceline-766054.TXT	9002.9	475.6	931.1	16	1302.6	-371.5	11	5	0.71	0.67
Average	**1015.6**	**45.1**	**72.7**	**48**	**115.6**	**-42.9**	**17**	**12**	**0.71**	**0.67**

Table 5.8: *

name - stock name, OT - profit of Omniscient trader, *B&H* - profit of Buy & Hold, Hurst - profit of tested strategy, Gains - Total gain, Lost - Total lost, Tg - no. of gain transactions, Tl - no. of lost transactions, Lt - level BUY, Lv - Level SELL

All result of DAX are in a table 5.9 and plot in figure 5.2. Same for a NASDAQ, table 5.11 and plot at figure 5.3. In tables and plots the results are compared with the prove of concept test.

Best results:

- test14 DAX

  average profit Omniscient trader: 1629.8

  average profit Buy& Hold: 31.6

  average profit MACD: 25.8

  average profit Hurst: 30.0

- test05 NASDAQ

  average profit Omniscient trader: 1015.6

  average profit Buy& Hold: 43.1

  average profit MACD: 7.5

  average profit Hurst: 38.3

5.1.3 MHCD

MHCD is Moving Hurst Convergence Divergence. To build this technical indicator we followed the mechanism of MACD, Moving Average Convergence Divergence technical indicator. One fast moving Hurst line and one slow moving Hurst line are crossing each other, generating BUY or SELL signal at each crossing. I experimented with different box size for a moving Hurst function and different return period. Right at the end of my testing I found out that positions of both lines (slow and fast) can be swapped and the indicator is still able to produce about three to four time better results than Buy & Hold strategy.

BUY signal is generated when the fast line crosses the slow line from up down. SELL signal is generated when the fast line crosses the slow line from down up.

All DAX results in a table 5.13 and a plot is at figure 5.4. Results for a NASDAQ are in table 5.15 and a plot in a figure 5.5. Detailed view of the BUY and SELL signals is shown on a plot of DAX Adidas stock plot figure 5.6 .

Best results:

- test27 DAX table 5.17 table 5.19

  average profit Omniscient trader: 1651.7

  average profit Buy& Hold: 41.2

  average profit MACD: 25.8

average profit MHCD: 186.2

- test28 NASDAQ table 5.23

  average profit Omniscient trader: 1010.4

  average profit Buy& Hold: 43.2

  average profit MACD: 7.5

  average profit MHCD: 125.7

5.2 Hypothesis testing

For a hypotesis testing the paired t-test statistics was used on results from Test27 DAX (table 5.17) and Test28 NASDAQ (table 5.23).

A paired t-test is used to compare two population means where you have two samples in which observations in one sample can be paired with observations in the other sample [20].

At paired t-test statistics profits from MHCD and MACD where tested against each other for all DAX and NASDAQ stocks.

5.2.1 MHCD vs MACD

Zero hypothesis state that profits from MHCD and MACD are the same. Alternative hypothesis is that profits from MHCD are higher. $H_0 : \mu_{MHCD} = \mu_{MACD}$
$H_A : \mu_{MHCD} > \mu_{MACD}$

5.2.1.1 Test27 DAX

```
>macd
 [1]  30.8  32.7  47.5  25.0  -2.3 -39.0  58.6  56.8  58.8  39.5   7.8  11.9
[13] -15.2 -19.5   3.2  24.0 -39.9  22.2  45.3  21.8  -9.5 -22.3  16.7  16.9
[25]   7.7  91.7  19.5 231.7

>mhcd
 [1] 133.9 568.3 114.6 154.2 144.5  81.1 396.7 206.1 171.8 255.1 185.5  39.1
[13]  67.4  80.6  39.1 191.3 264.2 108.6 104.2 224.0  67.3 142.7 458.0 170.1
[25] 213.1 330.3  82.6 218.2

> alpha = 0.01

> test = t.test(macd, y=mhcd, alternative='less', paired=TRUE, conf.level=1-alpha)

> test
```

```
Paired t-test

data:  macd and mhcd
t = -6.9046, df = 27, p-value = 1.014e-07
alternative hypothesis: true difference in means is less than 0
99 percent confidence interval:
      -Inf -102.9352
sample estimates:
mean of the differences
              -160.3643
```

I reject theory H_0 with possible error $\alpha = 1\%$. Winning theory is H_A, MHCD has a greater income rate than MACD.

5.2.1.2 Test28 NASDAQ

```
> macd
 [1]    7.3   -6.0  -31.1   84.9  -23.6   -6.2   -5.0   16.2   -2.9   18.0
[11]  -72.5  -68.3    5.2   12.0   -6.8  -17.1    4.8   21.1  -10.8   14.5
[21] -170.4    8.1   13.9    7.0   21.9   14.9   15.1    5.6  -30.2   -5.4
[31]  356.2  -25.3   63.9  -14.3   14.9  -53.1 -151.8   34.5    0.6   12.5
[41]   -8.4    2.0  -56.1   14.3  -12.0 -111.7  -29.5   34.4  -16.7  -13.8
[51]   15.1  110.5    4.0   -4.1  177.5   53.7   -9.0  -21.7  -10.0  131.6
[61]   39.1    6.5   78.8   45.3   15.5    5.9  -39.6   24.7   25.8  -11.9
[71]   -4.8   53.5  -41.5   12.2   18.9  -11.5    1.1   46.9  -56.4   -5.7
[81]  -64.6   10.7    9.0   82.7  -96.5   10.2   17.3  184.1

> mhcd
 [1]  40.7 149.4  77.4 253.2 244.1  95.4  33.9 127.4  50.9  46.5 226.2 118.0
[13] 180.3  55.1  44.5 161.7  51.7  61.8 108.7  51.0 330.1  44.0  62.3 108.0
[25] 136.8  18.6  56.1  74.5 132.4  63.6 459.8 114.6 170.6 150.9  67.2 720.1
[37] 212.6 107.3   8.5  23.0  81.9  27.8 184.4  86.6  88.9 159.9 146.0  91.2
[49]  28.9  55.2  76.8 456.7  88.2  44.9 297.1  88.8 104.3 148.0 119.4 296.7
[61]  70.6  50.7  90.7 166.5  43.0  66.2  51.3  61.0 112.5  59.3  -7.4 145.7
[73]  94.9  42.6  17.8  24.2  32.6 159.2 143.0   4.4  65.6 121.8  43.3 237.5
[85] 157.4 172.2 109.5 819.5

> alpha = 0.01

> test = t.test(macd, y=mhcd, alternative='less', paired=TRUE, conf.level=1-alpha)

> test
```

```
Paired t-test

data:  macd and mhcd
t = -8.8001, df = 87, p-value = 5.764e-14
alternative hypothesis: true difference in means is less than 0
99 percent confidence interval:
      -Inf -86.42111
sample estimates:
mean of the differences
              -118.2739
```

I reject theory H_0 with possible error $\alpha = 1\%$. Winning theory is H_A, MHCD has a greater income rate than MACD.

5.2.2 MHCD vs Buy & Hold

The same method was used to test MHCD results against Buy & Hold strategy. Zero hypothesis state that profits from MHCD and Buy & Hold have the same profits. Alternative hypothesis is that profits from MHCD are higher.
$H_0 : \mu_{MHCD} = \mu_{B\&H}$
$H_A : \mu_{MHCD} > \mu_{B\&H}$

5.2.2.1 Test27 DAX

```
> bh
 [1]   72.3   -6.1   59.3   62.4   69.3   56.1 -100.3  110.5   23.5    3.2
[11]   34.6   -1.2   -6.8    2.9   29.3  109.6   18.5   67.1   15.9  107.6
[21]    3.4   88.8   63.3   -4.6   47.9   58.0    5.6  164.8

> mhcd
 [1] 133.9 568.3 114.6 154.2 144.5  81.1 396.7 206.1 171.8 255.1 185.5  39.1
[13]  67.4  80.6  39.1 191.3 264.2 108.6 104.2 224.0  67.3 142.7 458.0 170.1
[25] 213.1 330.3  82.6 218.2

> alpha = 0.01

> test = t.test(bh, y=mhcd, alternative='less', paired=TRUE, conf.level=1-alpha)

> test

Paired t-test

data:  bh and mhcd
```

```
t = -5.449, df = 27, p-value = 4.568e-06
alternative hypothesis: true difference in means is less than 0
99 percent confidence interval:
      -Inf -79.15725
sample estimates:
mean of the differences
              -144.9179
```

I reject theory H_0 with possible error $\alpha = 1\%$. Winning theory is H_A, MHCD has a greater income rate than Buy & Hold.

5.2.2.2 Test28 NASDAQ

```
> bh
 [1]   31.8  105.4   39.7  201.7   13.5   -9.4   27.5   92.6   36.4   49.3
[11]   40.7   12.0   33.0   35.0   19.6   65.3   38.0   49.6    7.2   26.1
[21] -220.3   36.0   21.4   48.5   48.6   18.3   28.7   31.6   30.3   50.3
[31]  342.7   74.4   56.9    6.0   49.5  272.0   28.3   69.5    6.3    3.4
[41]   10.5    9.8   18.2   28.3  -11.1   15.8    1.1   22.7    0.3   47.3
[51]   25.1  240.9   11.7   33.9   16.9   86.2   33.5   15.5   55.9  191.3
[61]   25.4   45.5   54.7   55.8   17.3   53.7    1.9   47.0   71.4   13.3
[71]  -29.7  130.3   24.4   43.4   35.3    9.9   28.1    9.5   36.6   -5.8
[81]  -27.6   39.8   28.3  118.0   32.2   25.5   34.8  109.6

> mhcd
 [1]  40.7 149.4  77.4 253.2 244.1  95.4  33.9 127.4  50.9  46.5 226.2 118.0
[13] 180.3  55.1  44.5 161.7  51.7  61.8 108.7  51.0 330.1  44.0  62.3 108.0
[25] 136.8  18.6  56.1  74.5 132.4  63.6 459.8 114.6 170.6 150.9  67.2 720.1
[37] 212.6 107.3   8.5  23.0  81.9  27.8 184.4  86.6  88.9 159.9 146.0  91.2
[49]  28.9  55.2  76.8 456.7  88.2  44.9 297.1  88.8 104.3 148.0 119.4 296.7
[61]  70.6  50.7  90.7 166.5  43.0  66.2  51.3  61.0 112.5  59.3  -7.4 145.7
[73]  94.9  42.6  17.8  24.2  32.6 159.2 143.0   4.4  65.6 121.8  43.3 237.5
[85] 157.4 172.2 109.5 819.5

> alpha = 0.01

> test = t.test(bh, y=mhcd, alternative='less', paired=TRUE, conf.level=1-alpha)

> test

Paired t-test

data:  bh and mhcd
```

```
t = -6.9497, df = 87, p-value = 3.184e-10
alternative hypothesis: true difference in means is less than 0
99 percent confidence interval:
      -Inf -54.41402
sample estimates:
mean of the differences
              -82.57273
```

I reject theory H_0 with possible error $\alpha = 1\%$. Winning theory is H_A, MHCD has a greater income rate than Buy & Hold.

5.3 Results summary

Goal of this work was to develop and test new indicator for a technical analysis based on Hurst exponent measure. Using statistical tools we proved that new MHCD indicator presented in this work generates more profit compared to MACD technical indicator and Buy & Hold investment strategy. MHCD is 4.5 times better than Buy & Hold and 7.2 times better than MACD on DAX. On NASDAQ it's 2.9 times better than Buy & Hold and 16.8 times better than MACD.

Although It must be said that this profit is only theoretical. When we simulated real investment environment with 2000 in a wallet for a start and charged fee of 1% per a transaction (BUY or SELL), We finished with 624.4 in the wallet on DAX and ended up paying 7137.6 on fees. On NASDAQ We finished with 7819.8 and spent 34497.3 on fees. To make a picture clear, using Buy & Hold on DAX we'd end up with 8175.4 in my wallet and we'd pay 102.1 on fees. On NASDAQ with Buy & Hold we'd make 16648.1 and for fees I'd pay 187.7. There is no real profit to be made so far.

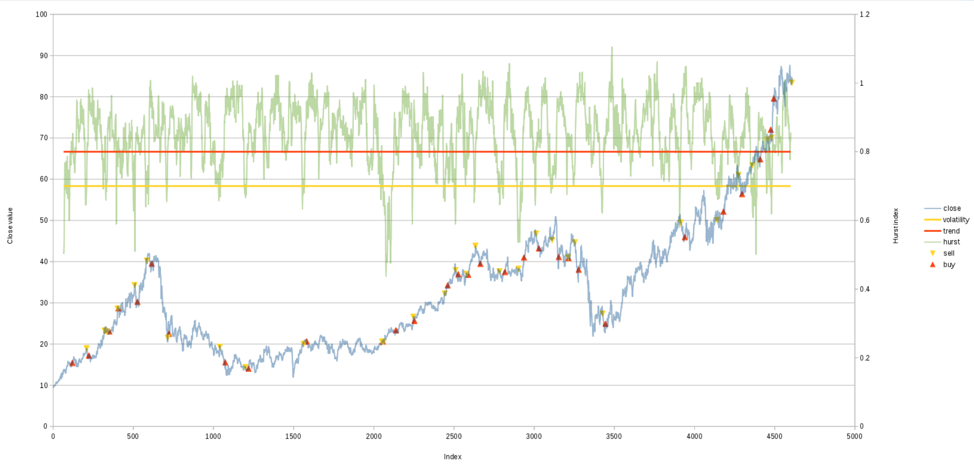

Figure 5.1: Test03 Adidas Hurst, levels trend and volatility

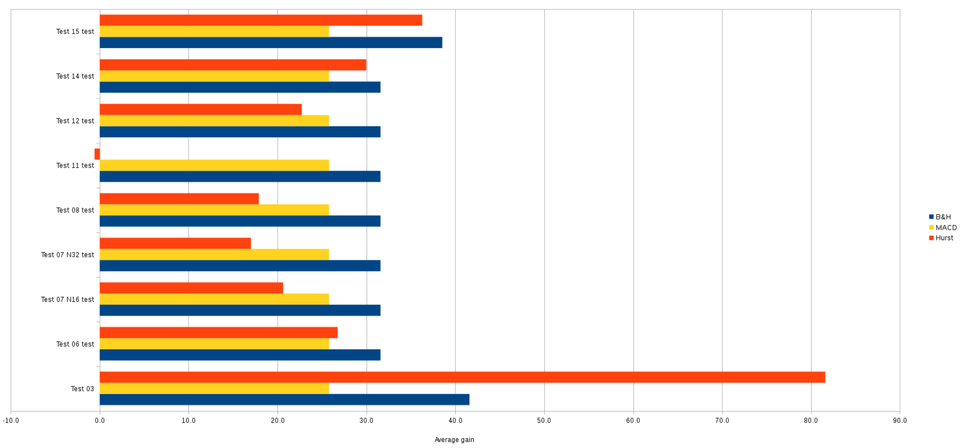

Figure 5.2: DAX Hurst teach and test

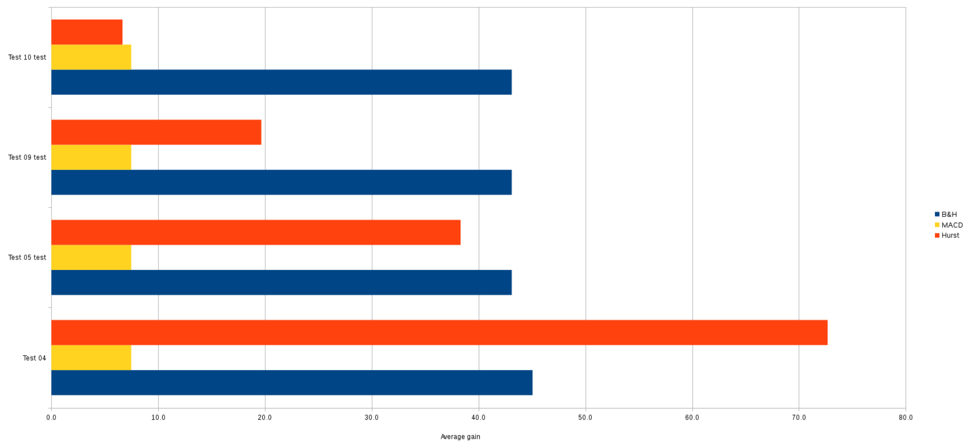

Figure 5.3: NASDAQ Hurst teach and test

Table 5.9: DAX teach and test

Market	**Name**	**OT**	**B&H**	**Hurst**	**MACD**	**T**	**Gain**	**Lost**	**Tg**	**Tl**
DAX	Test 03	1629.8	41.6	81.6	25.8	116	171.4	-89.8	25	17
DAX	Test 06 test	1629.8	31.6	26.8	25.8	31	85.4	-58.6	18	13
DAX	Test 07 N16 test	1629.8	31.6	20.6	25.8	19	49.4	-28.8	10	9
DAX	Test 07 N32 test	1629.8	31.6	17.0	25.8	12	34.1	-17.1	6	6
DAX	Test 08 test	1629.8	31.6	17.9	25.8	418	183.9	-166.0	208	210
DAX	Test 11 test	1629.8	31.6	-0.6	25.8	3	12.7	-13.3	1	2
DAX	Test 12 test	1629.8	31.6	22.7	25.8	115	97.6	-74.9	41	74
DAX	Test 14 test	1629.8	31.6	30.0	25.8	101	105.1	-71.4	29	73
DAX	Test 15 test	1629.8	38.5	36.3	25.8	212	199.9	-163.6	89	123

Table 5.10: *

name - stock name, OT - profit of Omniscient trader, *B&H* - profit of Buy & Hold, Hurst - profit of tested strategy, MACD - profit from MACD, T - no. of transactions, Gains - Total gain, Lost - Total lost, Tg - no. of gain transactions, Tl - no. of lost transactions

Table 5.11: NASDAQ teach and test

Market	**Name**	**OT**	**B&H**	**Hurst**	**MACD**	**T**	**Gain**	**Lost**	**Tg**	**Tl**
NASDAQ	Test 04	1015.6	45.1	72.7	7.5	48	115.6	-42.9	17	12
NASDAQ	Test 05 test	1015.6	43.1	38.3	7.5	19	61.8	-23.5	11	8
NASDAQ	Test 09 test	1015.6	43.1	19.7	7.5	202	94.3	-74.6	100	103
NASDAQ	Test 10 test	1015.6	43.1	6.7	7.5	4	9.1	-2.5	1	2

Table 5.12: *

name - stock name, OT - profit of Omniscient trader, *B&H* - profit of Buy & Hold, Hurst - profit of tested strategy, MACD - profit from MACD, T - no. of transactions, Gains - Total gain, Lost - Total lost, Tg - no. of gain transactions, Tl - no. of lost transactions

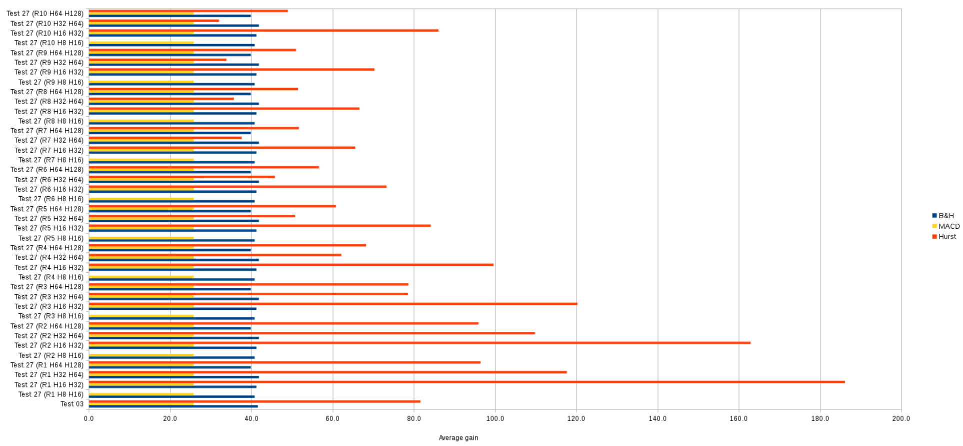

Figure 5.4: DAX Hurst different

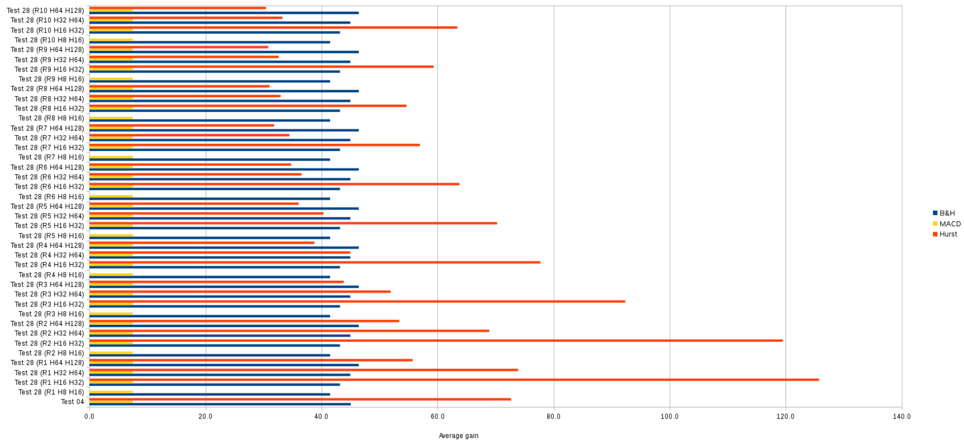

Figure 5.5: NASDAQ Hurst different

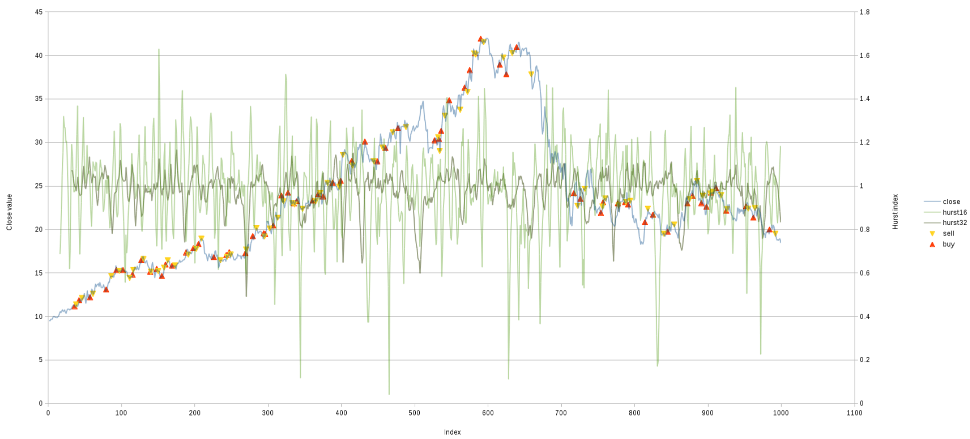

Figure 5.6: Test27 Adidas R1, H16, H32, detail

Table 5.13: DAX MHCD

Market	Name	OT	B&H	Hurst	MACD	T	Gain	Lost	Tg	Tl
DAX	Test 03	1629.8	41.6	81.6	25.8	116	171.4	-89.8	25	17
DAX	Test 27 (R1 H8 H16)	1629.8	40.8	0.0	25.8	1	0.0	0.0	0	1
DAX	Test 27 (R1 H16 H32)	1629.8	41.2	186.2	25.8	279	308.5	-122.3	182	97
DAX	Test 27 (R1 H32 H64)	1629.8	41.8	117.7	25.8	174	236.7	-119.0	111	63
DAX	Test 27 (R1 H64 H128)	1629.8	39.9	96.4	25.8	110	197.3	-100.9	70	40
DAX	Test 27 (R2 H8 H16)	1629.8	40.8	0.0	25.8	1	0.0	0.0	0	1
DAX	Test 27 (R2 H16 H32)	1629.8	41.2	162.9	25.8	267	297.3	-134.4	174	93
DAX	Test 27 (R2 H32 H64)	1629.8	41.8	109.8	25.8	169	230.3	-120.5	107	61
DAX	Test 27 (R2 H64 H128)	1629.8	39.9	95.9	25.8	108	194.1	-98.2	69	39
DAX	Test 27 (R3 H8 H16)	1629.8	40.8	0.0	25.8	1	0.0	0.0	0	1
DAX	Test 27 (R3 H16 H32)	1629.8	41.2	120.3	25.8	259	268.0	-147.8	161	98
DAX	Test 27 (R3 H32 H64)	1629.8	41.8	78.5	25.8	165	210.3	-131.8	99	66
DAX	Test 27 (R3 H64 H128)	1629.8	39.9	78.7	25.8	107	181.9	-103.2	65	41
DAX	Test 27 (R4 H8 H16)	1629.8	40.8	0.0	25.8	1	0.0	0.0	0	1
DAX	Test 27 (R4 H16 H32)	1629.8	41.2	99.6	25.8	253	253.8	-154.2	154	99
DAX	Test 27 (R4 H32 H64)	1629.8	41.8	62.2	25.8	161	198.2	-136.0	95	66
DAX	Test 27 (R4 H64 H128)	1629.8	39.9	68.2	25.8	105	174.3	-106.0	62	42
DAX	Test 27 (R5 H8 H16)	1629.8	40.8	0.0	25.8	1	0.0	0.0	0	1
DAX	Test 27 (R5 H16 H32)	1629.8	41.2	84.1	25.8	251	244.8	-160.7	150	101
DAX	Test 27 (R5 H32 H64)	1629.8	41.8	50.8	25.8	156	187.6	-136.8	91	65
DAX	Test 27 (R5 H64 H128)	1629.8	39.9	60.8	25.8	103	168.6	-107.8	61	42
DAX	Test 27 (R6 H8 H16)	1629.8	40.8	0.0	25.8	1	0.0	0.0	0	1
DAX	Test 27 (R6 H16 H32)	1629.8	41.2	73.3	25.8	249	237.5	-164.2	148	102
DAX	Test 27 (R6 H32 H64)	1629.8	41.8	45.8	25.8	154	183.6	-137.8	89	65
DAX	Test 27 (R6 H64 H128)	1629.8	39.9	56.6	25.8	101	164.9	-108.3	59	42
DAX	Test 27 (R7 H8 H16)	1629.8	40.8	0.0	25.8	1	0.0	0.0	0	1
DAX	Test 27 (R7 H16 H32)	1629.8	41.2	65.6	25.8	248	233.6	-168.1	146	103
DAX	Test 27 (R7 H32 H64)	1629.8	41.8	37.6	25.8	152	178.5	-140.9	88	64
DAX	Test 27 (R7 H64 H128)	1629.8	39.9	51.7	25.8	99	161.5	-109.9	58	41
DAX	Test 27 (R8 H8 H16)	1629.8	40.8	0.0	25.8	1	0.0	0.0	0	1
DAX	Test 27 (R8 H16 H32)	1629.8	41.2	66.6	25.8	249	232.6	-166.0	145	104
DAX	Test 27 (R8 H32 H64)	1629.8	41.8	35.7	25.8	151	177.6	-141.9	87	64
DAX	Test 27 (R8 H64 H128)	1629.8	39.9	51.5	25.8	98	159.9	-108.4	57	41
DAX	Test 27 (R9 H8 H16)	1629.8	40.8	0.0	25.8	1	0.0	0.0	0	1
DAX	Test 27 (R9 H16 H32)	1629.8	41.2	70.3	25.8	250	233.1	-162.8	145	105
DAX	Test 27 (R9 H32 H64)	1629.8	41.8	33.8	25.8	149	175.2	-141.4	87	63
DAX	Test 27 (R9 H64 H128)	1629.8	39.9	51.0	25.8	98	158.4	-107.4	57	40
DAX	Test 27 (R10 H8 H16)	1629.8	40.8	0.0	25.8	1	0.0	0.0	0	1
DAX	Test 27 (R10 H16 H32)	1629.8	41.2	86.1	25.8	253	241.9	-155.9	146	107
DAX	Test 27 (R10 H32 H64)	1629.8	41.8	32.0	25.8	149	174.6	-142.6	87	62
DAX	Test 27 (R10 H64 H128)	1629.8	39.9	49.0	25.8	97	156.5	-107.5	57	40

Table 5.14: *

name - stock name, OT - profit of Omniscient trader, *B&H* - profit of Buy & Hold, Hurst - profit of tested strategy, MACD - profit from MACD, T - no. of transactions, Gains - Total gain, Lost - Total lost, Tg - no. of gain transactions, Tl - no. of lost transactions

Table 5.15: NASDAQ MHCD

Market	Name	OT	B&H	Hurst	MACD	T	Gain	Lost	Tg	Tl
NASDAQ	Test 04	1015.6	45.1	72.7	7.5	48	115.6	-42.9	17	12
NASDAQ	Test 28 (R1 H8 H16)	1015.6	41.5	0.0	7.5	1	0.0	0.0	0	1
NASDAQ	Test 28 (R1 H16 H32)	1015.6	43.2	125.7	7.5	209	202.3	-76.6	135	73
NASDAQ	Test 28 (R1 H32 H64)	1015.6	45.0	73.9	7.5	127	144.6	-70.8	81	47
NASDAQ	Test 28 (R1 H64 H128)	1015.6	46.4	55.7	7.5	83	115.3	-59.6	52	31
NASDAQ	Test 28 (R2 H8 H16)	1015.6	41.5	0.0	7.5	1	0.0	0.0	0	1
NASDAQ	Test 28 (R2 H16 H32)	1015.6	43.2	119.5	7.5	199	197.3	-77.8	130	69
NASDAQ	Test 28 (R2 H32 H64)	1015.6	45.0	68.9	7.5	124	142.5	-73.6	79	45
NASDAQ	Test 28 (R2 H64 H128)	1015.6	46.4	53.4	7.5	82	114.9	-61.5	51	30
NASDAQ	Test 28 (R3 H8 H16)	1015.6	41.5	0.0	7.5	1	0.0	0.0	0	1
NASDAQ	Test 28 (R3 H16 H32)	1015.6	43.2	92.3	7.5	193	178.1	-85.7	119	73
NASDAQ	Test 28 (R3 H32 H64)	1015.6	45.0	51.9	7.5	121	130.8	-78.8	72	48
NASDAQ	Test 28 (R3 H64 H128)	1015.6	46.4	43.8	7.5	80	109.3	-65.4	48	33
NASDAQ	Test 28 (R4 H8 H16)	1015.6	41.5	0.0	7.5	1	0.0	0.0	0	1
NASDAQ	Test 28 (R4 H16 H32)	1015.6	43.2	77.7	7.5	188	167.9	-90.2	114	74
NASDAQ	Test 28 (R4 H32 H64)	1015.6	45.0	45.0	7.5	118	125.3	-80.3	69	49
NASDAQ	Test 28 (R4 H64 H128)	1015.6	46.4	38.8	7.5	79	105.5	-66.7	46	33
NASDAQ	Test 28 (R5 H8 H16)	1015.6	41.5	0.0	7.5	1	0.0	0.0	0	1
NASDAQ	Test 28 (R5 H16 H32)	1015.6	43.2	70.3	7.5	186	163.0	-92.7	111	75
NASDAQ	Test 28 (R5 H32 H64)	1015.6	45.0	40.3	7.5	115	120.8	-80.5	67	48
NASDAQ	Test 28 (R5 H64 H128)	1015.6	46.4	36.1	7.5	78	103.4	-67.3	45	33
NASDAQ	Test 28 (R6 H8 H16)	1015.6	41.5	0.0	7.5	1	0.0	0.0	0	1
NASDAQ	Test 28 (R6 H16 H32)	1015.6	43.2	63.8	7.5	185	158.4	-94.6	109	76
NASDAQ	Test 28 (R6 H32 H64)	1015.6	45.0	36.6	7.5	113	117.7	-81.1	65	48
NASDAQ	Test 28 (R6 H64 H128)	1015.6	46.4	34.8	7.5	77	101.7	-66.9	44	33
NASDAQ	Test 28 (R7 H8 H16)	1015.6	41.5	0.0	7.5	1	0.0	0.0	0	1
NASDAQ	Test 28 (R7 H16 H32)	1015.6	43.2	57.0	7.5	185	156.0	-99.0	108	78
NASDAQ	Test 28 (R7 H32 H64)	1015.6	45.0	34.5	7.5	112	115.3	-80.8	64	48
NASDAQ	Test 28 (R7 H64 H128)	1015.6	46.4	31.8	7.5	76	99.8	-67.9	43	33
NASDAQ	Test 28 (R8 H8 H16)	1015.6	41.5	0.0	7.5	1	0.0	0.0	0	1
NASDAQ	Test 28 (R8 H16 H32)	1015.6	43.2	54.6	7.5	185	155.9	-101.3	107	78
NASDAQ	Test 28 (R8 H32 H64)	1015.6	45.0	33.0	7.5	111	114.1	-81.1	63	47
NASDAQ	Test 28 (R8 H64 H128)	1015.6	46.4	31.1	7.5	75	98.6	-67.5	43	32
NASDAQ	Test 28 (R9 H8 H16)	1015.6	41.5	0.0	7.5	1	0.0	0.0	0	1
NASDAQ	Test 28 (R9 H16 H32)	1015.6	43.2	59.3	7.5	186	157.6	-98.2	107	79
NASDAQ	Test 28 (R9 H32 H64)	1015.6	45.0	32.6	7.5	110	113.3	-80.6	63	46
NASDAQ	Test 28 (R9 H64 H128)	1015.6	46.4	30.8	7.5	75	98.2	-67.3	43	32
NASDAQ	Test 28 (R10 H8 H16)	1015.6	41.5	0.0	7.5	1	0.0	0.0	0	1
NASDAQ	Test 28 (R10 H16 H32)	1015.6	43.2	63.4	7.5	189	160.4	-97.0	108	80
NASDAQ	Test 28 (R10 H32 H64)	1015.6	45.0	33.3	7.5	109	113.1	-79.8	63	46
NASDAQ	Test 28 (R10 H64 H128)	1015.6	46.4	30.4	7.5	74	96.7	-66.3	42	31

Table 5.16: *

name - stock name, OT - profit of Omniscient trader, $B\&H$ - profit of Buy & Hold, Hurst - profit of tested strategy, MACD - profit from MACD, T - no. of transactions, Gains - Total gain, Lost - Total lost, Tg - no. of gain transactions, Tl - no. of lost transactions

Table 5.17: Test 27 DAX R1, H16, H32

Name	**OT**	**B&H**	**Hurst**	**T**	**Gain**	**Lost**	**Tg**	**Tl**
Adidas-A1EWWW.txt	1044.4	72.3	133.9	294	206.8	-72.9	193	101
Allianz-840400.txt	5360.8	-6.1	568.3	268	945.7	-377.4	174	94
BASF-BASF11.txt	964.3	59.3	114.6	310	193.0	-78.3	206	104
BMW-519000.txt	1313.7	62.4	154.2	283	240.4	-86.2	176	107
Bayer-BAY001.txt	1287.5	69.3	144.5	289	236.1	-91.6	198	91
Beiersdorf-520000.txt	1031.0	56.1	81.1	271	175.4	-94.3	162	109
Commerzbank-0BK100.txt	4265.7	-100.3	396.7	289	738.0	-341.3	178	111
Continental-543900.txt	1620.7	110.5	206.1	288	326.3	-120.2	187	101
Daimler-710000.txt	1673.4	23.5	171.8	279	320.0	-148.2	184	95
Dt-Bank-514000.txt	1950.4	3.2	255.1	284	391.4	-136.3	197	87
Dt-Boerse-581005.txt	1172.2	34.6	185.5	193	246.1	-60.6	131	62
Dt-Post-555200.txt	344.6	-1.2	39.1	219	64.1	-25.0	144	75
Dt-Telekom-555750.txt	706.6	-6.8	67.4	259	128.3	-60.9	167	92
EON-ENAG99.txt	630.0	2.9	80.6	312	132.7	-52.1	211	101
FMC-578580.txt	835.3	29.3	39.1	258	122.0	-82.9	147	111
Fresenius-578563.txt	1379.6	109.6	191.3	280	272.8	-81.5	190	90
Heidelberger-604700.txt	1820.9	18.5	264.2	288	369.7	-105.6	193	95
Henkel-604843.txt	846.7	67.1	108.6	306	175.6	-67.0	192	114
KaS-0SAG88.txt	798.3	15.9	104.2	286	162.6	-58.5	191	95
Linde-648300.txt	1957.1	107.6	224.0	282	347.2	-123.1	182	100
Lufthansa-823212.txt	551.9	3.4	67.3	268	105.7	-38.4	182	86
Merck-659990.txt	1699.5	88.8	142.7	287	297.9	-155.2	183	104
Muenchener-843002.txt	4966.8	63.3	458.0	269	837.2	-379.2	178	91
RWE-703712.txt	1363.5	-4.6	170.1	292	272.8	-102.7	179	113
SAP-716460.txt	1356.5	47.9	213.1	280	291.8	-78.7	191	89
Siemens-723610.txt	2202.7	58.0	330.3	297	473.7	-143.5	206	91
Thyssen-750000.txt	797.6	5.6	82.6	293	150.7	-68.1	196	97
Volkswagen-766403.txt	2304.7	164.8	218.2	299	413.1	-194.9	191	108
Average	**1651.7**	**41.2**	**186.2**	**279**	**308.5**	**-122.3**	**182**	**97**

Table 5.18: *

name - stock name, OT - profit of Omniscient trader, *B&H* - profit of Buy & Hold, Hurst - profit of tested strategy, T - no. of transactions, Gains - Total gain, Lost - Total lost, Tg - no. of gain transactions, Tl - no. of lost transactions

Table 5.19: Test 27 DAX index R1, H16, H32

Name	**OT**	**B&H**	**Hurst**	**T**	**Gain**	**Lost**	**Tg**	**Tl**
DAX-846900.txt	132827.9	5710.9	17943.1	280	26432.8	-8489.7	201	79

Table 5.20: *

name - stock name, OT - profit of Omniscient trader, *B&H* - profit of Buy & Hold, Hurst - profit of tested strategy, T - no. of transactions, Gains - Total gain, Lost - Total lost, Tg - no. of gain transactions, Tl - no. of lost transactions

Table 5.21: Test28 NASDAQ R1, H16, H32 part 1

Name	OT	B&H	Hurst	T	Gain	Lost	Tg	Tl
Avago-A0X9TN.TXT	237.0	31.8	40.7	80	51.6	-10.9	53	27
Baidu-A0F5DE.TXT	1193.0	105.4	149.4	152	247.5	-98.0	94	58
Bed-Bath-884304.TXT	1101.7	39.7	77.4	252	156.5	-79.2	152	100
Biogen-789617.TXT	2340.4	201.7	253.2	271	464.6	-211.4	168	103
Broadcom-913684.TXT	1861.2	13.5	244.1	241	359.4	-115.3	154	87
CA-Techn-A0JC59.TXT	1109.2	-9.4	95.4	286	198.6	-103.2	177	109
Catamaran-A1J08W.TXT	331.0	27.5	33.9	83	59.0	-25.1	51	32
Celgene-881244.TXT	1099.0	92.6	127.4	249	219.2	-91.8	156	93
Cerner-892807.TXT	389.2	36.4	50.9	241	80.6	-29.7	163	78
Charter-Comm-A0YF1T.TXT	251.9	49.3	46.5	32	57.7	-11.2	21	11
Check-Point-901638.TXT	1628.5	40.7	226.2	267	312.8	-86.5	177	90
Cisco-878841.TXT	880.2	12.0	118.0	282	169.3	-51.3	187	95
Citrix-898407.TXT	1562.2	33.0	180.3	277	298.8	-118.5	183	94
Cognizant-915272.TXT	383.7	35.0	55.1	242	80.4	-25.3	163	79
Comcast-157484.TXT	362.7	19.6	44.5	179	65.1	-20.6	122	57
Costco-888351.TXT	1382.0	65.3	161.7	253	254.8	-93.1	170	83
DirecTV-A1J1EZ.TXT	409.8	38.0	51.7	159	81.3	-29.6	101	58
Discovery-Comm-A0Q90G.TXT	407.4	49.6	61.8	146	93.4	-31.6	93	53
Dish-Network-A0NBN0.TXT	904.0	7.2	108.7	214	175.6	-66.9	137	77
Doller-Tree-A0NFQC.TXT	425.5	26.1	51.0	239	84.9	-33.9	155	84
Equinix-165241.TXT	2309.4	-220.3	330.1	215	483.1	-153.0	154	61
Expedia-A1JRLJ.TXT	497.0	36.0	44.0	128	100.3	-56.3	85	43
Expeditors-875272.TXT	722.4	21.4	62.3	230	123.7	-61.4	139	91
Express-Scripts-A1JWJL.TXT	588.8	48.5	108.0	247	134.9	-26.9	175	72
F5-922977.TXT	1595.7	48.6	136.8	239	300.8	-164.1	153	86
Facebook-A1JWVX.TXT	131.9	18.3	18.6	32	27.4	-8.7	19	13
Fastenal-887891.TXT	449.3	28.7	56.1	213	90.3	-34.1	141	72
Fiserv-881793.TXT	524.6	31.6	74.5	253	106.4	-31.9	179	74
Garmin-A1C06B.TXT	848.6	30.3	132.4	213	178.8	-46.5	148	65
Gilead-885823.TXT	432.9	50.3	63.6	291	87.0	-23.4	201	90
Google-A0B7FY.TXT	3331.6	342.7	459.8	175	766.0	-306.2	112	63
Henry-Schein-897961.TXT	904.5	74.4	114.6	251	184.3	-69.7	164	87
Illumina-927079.TXT	1112.0	56.9	170.6	229	259.5	-88.9	150	79
Intel-855681.TXT	950.0	6.0	150.9	271	208.7	-57.8	179	92
Intuit-886053.TXT	939.5	49.5	67.2	257	142.9	-75.7	163	94
Intuitivr-Surgical-888024.TXT	4557.7	272.0	720.1	223	1118.7	-398.6	146	77
KLA-Tencor-865884.TXT	1730.7	28.3	212.6	250	322.8	-110.3	162	88
Keurig-A1XFME.TXT	683.0	69.5	107.3	216	168.4	-61.1	140	76
Kraft-Foods-A1J20U.TXT	69.5	6.3	8.5	26	12.5	-4.0	19	7
Liberty-A0JMPL.TXT	167.9	3.4	23.0	128	38.3	-15.4	77	51
Liberty-Global-A1W0FL.TXT	525.8	10.5	81.9	156	120.4	-38.5	110	46
Liberty-Media-Corp-A1KBFW.TXT	191.4	9.8	27.8	15	40.6	-12.8	11	4
Linear-872629.TXT	1208.1	18.2	184.4	252	293.2	-108.8	171	81
Marriott-913070.TXT	682.1	28.3	86.6	239	129.1	-42.5	156	83
Mattel-851704.TXT	648.9	-11.1	88.9	261	137.2	-48.3	174	87
Maxim-Integrated-876158.TXT	1413.5	15.8	159.9	253	273.5	-113.6	162	91
Micron-869020.TXT	1165.3	1.1	146.0	272	207.0	-61.0	184	88
Microsoft-870747.TXT	797.6	22.7	91.2	270	144.0	-52.8	179	91
Mondelez-A1J4U0.TXT	317.9	0.3	28.9	199	56.9	-28.0	129	70
Monster-Beverage-A1JSKK.TXT	539.6	47.3	55.2	168	108.1	-52.9	111	57
Mylan-868270.TXT	454.0	25.1	76.8	245	103.7	-26.9	167	78

Table 5.22: *

name - stock name, OT - profit of Omniscient trader, $B\&H$ - profit of Buy & Hold, Hurst - profit of tested strategy, T - no. of transactions, Gains - Total gain, Lost - Total lost, Tg - no. of gain transactions, Tl - no. of lost transactions

Table 5.23: Test28 NASDAQ R1, H16, H32 part 2

Name	**OT**	**B&H**	**Hurst**	**T**	**Gain**	**Lost**	**Tg**	**Tl**
NETFlix-552484.TXT	2158.5	240.9	456.7	188	589.6	-133.0	132	56
NVIDIA-918422.TXT	536.9	11.7	88.2	237	119.2	-31.0	158	79
NXP-Semiconductors-A1C5WJ.TXT	232.4	33.9	44.9	61	53.6	-8.7	39	22
NetApp-A0NHKR.TXT	1759.3	16.9	297.1	234	389.4	-92.3	152	82
OReilly-A1H5JY.TXT	683.6	86.2	88.8	197	139.5	-50.7	120	77
PACCAR-861114.TXT	869.2	33.5	104.3	230	166.5	-62.2	143	87
Paychex-868284.TXT	971.5	15.5	148.0	271	203.1	-55.2	183	88
Qualcomm-883121.TXT	1299.3	55.9	119.4	267	228.6	-109.2	185	82
Regeneron-Pharma-881535.TXT	1725.8	191.3	296.7	224	428.9	-132.2	143	81
Robinson-Worldwide-A0HGF5.TXT	679.3	25.4	70.6	171	132.2	-61.6	118	53
Ross-870053.TXT	475.9	45.5	50.7	256	89.7	-39.0	154	102
SBA-Comm-923376.TXT	853.3	54.7	90.7	226	161.9	-71.2	149	77
Sandisk-897826.TXT	1403.3	55.8	166.5	269	270.4	-103.9	163	106
Seagate-A1C08F.TXT	468.6	17.3	43.0	162	83.5	-40.5	103	59
Sigma-Aldrich-863120.TXT	763.5	53.7	66.2	254	124.5	-58.4	156	98
Staples-876951.TXT	571.7	1.9	51.3	249	93.9	-42.7	158	91
Starbucks-884437.TXT	630.5	47.0	61.0	263	113.4	-52.4	163	100
Stericycle-902518.TXT	811.2	71.4	112.5	223	167.2	-54.7	159	64
Symantec-879358.TXT	453.8	13.3	59.3	279	87.8	-28.6	180	99
Syrius-A1W8XE.TXT	598.3	-29.7	-7.4	270	83.3	-90.7	153	117
Tesla-A1CX3T.TXT	682.1	130.3	145.7	64	171.4	-25.7	46	18
Texas-Instruments-852654.TXT	1269.3	24.4	94.9	268	193.4	-98.5	174	94
Tractor-889826.TXT	358.4	43.4	42.6	179	72.0	-29.4	120	59
TripAdvisor-A1JRLK.TXT	248.1	35.3	17.8	37	42.3	-24.6	25	12
Twenty-First-Century-A1WZP6.TXT	289.2	9.9	24.2	187	50.2	-25.9	110	77
Verisk-A0YA2M.TXT	160.0	28.1	32.6	68	41.3	-8.8	43	25
Vertex-882807.TXT	1442.8	9.5	159.2	211	298.5	-139.2	127	84
Viacom-A0HM1Q.TXT	1437.5	36.6	143.0	268	266.8	-123.8	160	108
VimpelCom-A0YE2R.TXT	88.9	-5.8	4.4	55	13.1	-8.8	32	23
Vodafon-A1XD9Z.TXT	660.5	-27.6	65.6	230	117.0	-51.4	143	87
Western-Digital-863060.TXT	888.5	39.8	121.8	266	190.5	-68.7	178	88
Whole-Foods-886391.TXT	542.2	28.3	43.3	244	90.5	-47.2	145	99
Wynn-Resorts-663244.TXT	1777.6	118.0	237.5	161	380.1	-142.6	103	58
Xilinx-880135.TXT	1474.1	32.2	157.4	290	256.2	-98.8	177	113
Yahoo-900103.TXT	1129.3	25.5	172.2	264	238.8	-66.6	171	93
eBay-916529.TXT	909.6	34.8	109.5	233	174.9	-65.4	157	76
priceline-766054.TXT	8861.9	109.6	819.5	243	1503.8	-684.3	161	82
Average	**1010.4**	**43.2**	**125.7**	**209**	**202.3**	**-76.6**	**135**	**73**

Table 5.24: *

name - stock name, OT - profit of Omniscient trader, $B\&H$ - profit of Buy & Hold, Hurst - profit of tested strategy, T - no. of transactions, Gains - Total gain, Lost - Total lost, Tg - no. of gain transactions, Tl - no. of lost transactions

Chapter 6

Conclusion

We followed the Fractal Markets Hypothesis, tried and succeeded in building up indicator for a technical analysis based on Hurst's Rescale Range Analysis MHCD, that proved to be better than widely known and used MACD and Buy & Hold strategies. Fractal Markets Hypothesis and it's interpretation of a market as a fractal system with local randomness and global determinism is offering new ways of seeing the markets and much of the natural systems. Lot more is yet to discoverer, but there is no doubt that fractals are closer to the world we're living in than any other concept, that is trying to hide unpleasant details behind nice and easy to use simplifications.

Bibliography

[1] https://en.wikipedia.org/wiki/Hurst_exponent

[2] https://www.cfainstitute.org/

[3] https://www.investopedia.com/university/technical/techanalysis1.asp

[4] https://www.investopedia.com/university/technical/#ixzz53QsQPKLU

[5] Barnesly, M. Fractals Everyvhere. San Diego, CA: Academic press, 1988

[6] http://www.bearcave.com/misl/misl_tech/wavelets/hurst/

[7] A.A.Anis, E.H.Lloyd (1976) The expected value of the adjusted rescaled Hurst range of independent normal summands, Biometrica 63, 283-298

[8] H.E.Hurst (1951) Long-term storage capacity of reservoirs, Transactions of the American Society of Civil Engineers 116, 770-808

[9] E.E.Peters (1994) Fractal Market Analysis, Wiley

[10] R.Weron (2002) Estimating long range dependence: finite sample properties and confidence intervals, Physica A 312, 285-299

[11] Illustration by Liv Feder from Fractals by Jens Feder, 1988

[12] http://www.onlinetradingconcepts.com/TechnicalAnalysis/MACD.html

[13] A Non-Random Walk Down Wall Street

[14] http://www.investopedia.com/terms/t/technicalanalysis.asp

[15] https://www.mathworks.com/help/curvefit/smooth.html

[16] By Beojan Stanislaus, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=8862246

[17] Benoit B. Mandelbrot (1983). The fractal geometry of nature. Macmillan. ISBN 978-0-7167-1186-5. Retrieved 1 February 2012.

[18] http://math.bu.edu/DYSYS/chaos-game/node6.html

[19] Brenner, Menachem, Fand Galai, Dan. New Financial Instruments for Hedging Changes in Volatility, Financial Analysts Journal, July/August 1989, http://people.stern.nyu.edu/mbrenner/research/FAJ_articleon_Volatility_Der.pdf

[20] http://www.statstutor.ac.uk/resources/uploaded/paired-t-test.pdf

[21] Safonov, Leonid A.; Tomer, Elad; Strygin, Vadim V.; Ashkenazy, Yosef; Havlin, Shlomo (2002). "Multifractal chaotic attractors in a system of delay-differential equations modeling road traffic". Chaos: An Interdisciplinary Journal of Nonlinear Science. 12 (4): 1006.

[22] Lorenz, Edward N. (1963). "Deterministic non-periodic flow". Journal of the Atmospheric Sciences. 20 (2): 130–141.

[23] Boeing (2015). "Chaos Theory and the Logistic Map". Retrieved 2015-07-16.

[24] By Jashuah - Own work by uploader, data from Chicago Board Options Exchange, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=19687514

[25] return: definition of return in Oxford dictionary (British & World English) https://en.oxforddictionaries.com/definition/return

[26] Volatility Definition http://www.investopedia.com/terms/v/volatility.asp

[27] http://www.investopedia.com/terms/b/buyandhold.asp

[28] Hurst, H.E. (1951). "Long-term storage capacity of reservoirs". Transactions of American Society of Civil Engineers. 116: 770.

[29] Hurst, H.E.; Black, R.P.; Simaika, Y.M. (1965). Long-term storage: an experimental study. London: Constable.

[30] Mandelbrot, B.B.; Wallis, J.R. (1968). "Noah, Joseph, and operational hydrology". Water Resour. Res. 4: 909–918. doi:10.1029/wr004i005p00909.

[31] Mandelbrot, Benoît B. "The (Mis)Behavior of Markets": 187.

[32] orsten Kleinow (2002)Testing Continuous Time Models in Financial Markets, Doctoral thesis, Berlin

[33] Qian, Bo; Rasheed, Khaled (2004). HURST EXPONENT AND FINANCIAL MARKET PREDICTABILITY. IASTED conference on Financial Engineering and Applications (FEA 2004). pp. 203–209

[34] Feder, Jens (1988). Fractals. New York: Plenum Press. ISBN 0-306-42851-2

[35] Mandelbrot, Benoit B. (1985). "Self-affinity and fractal dimension". Physica Scripta. 32: 257–260

[36] Mandelbrot, Benoit B.; Wallis, James R. (1969-10-01). "Robustness of the rescaled range R/S in the measurement of noncyclic long run statistical dependence". Water Resources Research. 5 (5): 967–988.

[37] Geweke, J.; Porter-Hudak, S. (1983). J. Time Ser. Anal. 4: 221

[38] J. Beran. Statistics For Long-Memory Processes. Chapman and Hall, 1994

[39] Robinson, P. M. (1995). "Gaussian semiparametric estimation of long-range dependence". The Annals of Statistics. 23: 1630–1661

[40] Simonsen, Ingve; Hansen, Alex; Nes, Olav Magnar (1998-09-01). "Determination of the Hurst exponent by use of wavelet transforms". Physical Review E. 58 (3): 2779–2787

[41] R. H. Riedi. Multifractal processes. In P. Doukhan, G. Oppenheim, and M. S. Taqqu, editors, The- ory And Applications Of Long-Range Dependence, pages 625–716. Birkhauser, 2003

[42] Aaron Clauset; Cosma Rohilla Shalizi; M. E. J. Newman (2009). "Power-law distributions in empirical data". SIAM Review. 51: 661–703

[43] Annis, A. A.; Lloyd, E. H. (1976-01-01). "The expected value of the adjusted rescaled Hurst range of independent normal summands". Biometrika. 63 (1): 111–116

[44] Bauwens, L., Hafner, Ch., Laurent, S.: Handbook of Volatility Models and their Applications. Wiley, Published Online, pp. 487–535, 2012

[45] Brooks, Ch., Persand, G.: Volatility forecasting for risk management. Journal of Forecasting, Volume 22, Issue 1, pp. 1–22, January 2003

[46] Caccia, D.C., Percival, D., Cannon, M.J. Raymond, G., and Bassingthwaighte, J.B.: Analyzing exact fractal time series: evaluating dispersional analysis and rescaled range methods. Physica A., December 1, 246(3-4), pp .609 -– 632, 1997.

[47] Conover, J.: Notes on the Fractal Analysis of Various Market Segments in the North American Electronics Industry. 2006 http://www.johncon.com/ndustrix/archive/fractal.pdf

[48] Falconer, K.: Fractal Geometry: Mathematical Foundations and Applications. Wiley, Chichester, 1990. MR1102677

[49] Fama, E.: Efficient capital markets: A review of theory and empirical work. Journal of Finance 25, 383–417 (1970)

[50] Figlewski, S.: Forecasting Volatility. In: Financial Markets, Institutions and Instruments, Volume 6, Issue 1, pp. 1–88, February 1997.

[51] Gneiting, T. and Schlather, M.: Stochastic models that separate fractal dimension and the Hurst effect. SIAM Rev. 46, pp. 269—282 (electronic), 2004. MR2114455

[52] Gneiting, T., Sevcikova, H. and Percival, D.B.: Estimators of Fractal Dimension: Assessing the Roughness of Time Series and Spatial Data. Statistical Science 2012, Vol. 27, No. 2, pp. 247—277, 2012.

[53] Goldstein, D. G., Taleb, N. N.: We don't quite know what we are talking about when we talk about volatility. Journal of Portfolio Management, 33(4), 84-86, 2007.

[54] Gospodinov, N., Gavala, A., Jiang, D.: Forecasting Volatility. Journal of Forecasting 25, pp. 381–400, Published online in Wiley InterScience, 2006.

[55] Halley, J. M., Hartley, S., Kallimanis, A. S., Kunin, W. E., Lennon, J. J. and Sgardelis, S. P.: Uses and abuses of fractal methodology in ecology. Ecology Letters 7, pp. 254—271, 2004.

[56] Hausdorff, F.: Dimension und äußeres Maß. Math. Ann. 79, pp. 157 — 179, 1919.

[57] Hull, J.C.: Options, Futures and Other Derivatives, Eighth Edition, 2012

[58] Hurst, H.E.: The Long-Term Storage Capacity of Reservoirs. Transaction of th eAmerican Society of Civil Engineers 116, 1951.

[59] Kou, S.G.: Lévy processes in Asset Pricing. Columbia University, http://www.columbia.edu/ sk75/levy.pdf, 2007.

[60] Kroha, P., Lauschke, M.: Fuzzy and Fractal Technology in Market Analysis. In: Madani, K., Correira, A.D., Rosa, A., Filipe, J. (Eds.): Computational Inteligence, Revised and Selected Papers of the International Joint Conference, IJCCI 2010, Studies in Computational Intelligence, Volume 399, pp. 247-260, Springer 2012.

[61] Liu, S.: An improved differential box-counting approach to compute fractal dimension of gray-level image. International Symposium on Information Science and Engineering, IEEE, 2008.

[62] Lo, A.W.: Long-term memory in stock market prices. Econometrica 59, pp. 1279–1313, 1991.

[63] Lo, A.W., MacKinlay, A.C.: A Non-Random Walk Down Wall Street. Princeton University Press, 1999.

[64] Mandelbrot, B.: Fractals and Scaling in Finance. Springer Verlag, 1997.

[65] Malcai, O., Lidar, D. A., Biham, O. and Avnir, D.: Scaling range and cutoffs in empirical fractals. Phys. Rev. E 56 pp. 2817–2828, 1997.

[66] Malkiel, B.: A Random Walk Down Wall Street. W.W. Norton, New York, 1996.

[67] Mesa, O.J., Poveda, G.: The Hurst Effect: The Scale of Fluctuation Approach. Water Resources Research, Vol. 29, No. 12, pp. 3995 – 4002, 1993.

[68] Peters, E.E.: Fractal Market Analysis - Applying Chaos Theory to Investment and Economics. John Wiley and Sons, 1994.

[69] Peters, E.E.: Chaos and Order in the Capital Markets. Second Edition, John Wiley and Sons, 1996.

[70] Feder, J.: Fractals. Plenum Press, 1988.

[71] Shleifer, A.: Inefficient Markets – An Introduction to Behavioral Finance. Oxford University Press, 2000.

[72] Termonia, Y., Alexandrowicz, Z.: Fractal Dimension of Strange Attractors from Radius Versus Size of Arbitrary Clusters. Phys. Rev. Lett. 51, 1265, 1983.

[73] TEVEROVSKY, Vadim; TAQQU, Murad S.; WILLINGER, Walter. A critical look at Lo's modified R/S statistic. Journal of statistical Planning and Inference, 1999, 80.1: 211-227.

[74] Theiler, J.: Estimating the Fractal Dimension of Chaotic Time Series. The Lincoln Laboratory Journal, Volume 3, Number 1, 1990.

[75] Velasquez, T.: Chaos Theory and the Science of Fractals, and their Application in Risk Management. Copenhagen Business School, 2009.

[76] Weron, R.: Estimating long range dependence: finite sample properties and confidence intervals. Physica A 312, pp.285 - 299, 2002.

[77] Realized Volatility Forecast of Stock Index Under Structural Breaks JOURNAL OF FORECASTING Ke Yang, Langnan Chen and Fengping Tian Article first published online : 28 NOV 2014

Appendix A

List of Abbreviations

CMT Capital market theory

csv Comma-separated values

EMH Efficient market hypothesis

FMH Fractal market hypothesis

MACD Moving Average Convergence Divergence

MHCD Moving Hurst Convergence Divergence

XML Extensible markup language

Appendix B

Content SD card

- doc .... test result summaries and info text
  - thesis.pdf .... thesis in PDF
- paper .... source files of thesis in $\LaTeX$
- program
  - data .... test data and scripts to clear the data
  - results .... test results, raw data
  - src .... aplication source files in MATLAB

Appendix C

Used data

C.0.1 DAX

For simulations I used daily close price of following stocks from DAX index:

1. Adidas-A1EWWW from 27-Dec-1995 to 23-Aug-2013
2. Allianz-840400 from 17-Jul-1995 to 23-Aug-2013
3. BASF-BASF11 from 17-Jul-1995 to 20-Aug-2013
4. Bayer-BAY001 from 17-Jul-1995 to 20-Aug-2013
5. Beiersdorf-520000 - from 25-Jul-1996 to 23-Aug-2013
6. BMW-519000 from 17-Jul-1995 to 23-Aug-2013
7. Commerzbank-0BK100 from 17-Jul-1995 to 05-Sep-2013
8. Continental-543900 from 17-Jul-1995 to 23-Aug-2013
9. Daimler-710000 from 17-Jul-1995 to 23-Aug-2013
10. Dt-Bank-514000 from 17-Jul-1995 to 23-Aug-2013
11. Dt-Boerse-581005 from 01-Jun-2001 to 23-Aug-2013
12. Dt-Post-555200 from 20-Nov-2000 to 23-Aug-2013
13. Dt-Telekom-555750 from 18-Nov-1996 to 23-Aug-2013
14. EON-ENAG99 from 17-Jul-1995 to 20-Aug-2013
15. FMC-578580 from 10-Oct-1996 to 24-Feb-2014
16. Fresenius-578563 from 17-Jul-1995 to 24-Feb-2014

17. Heidelberger-604700 from 17-Jul-1995 to 23-Aug-2013
18. Henkel-604843 from 17-Jul-1995 to 23-Aug-2013
19. KaS-0SAG88 from 17-Jul-1995 to 23-Aug-2013
20. Linde-648300 from 17-Jul-1995 to 23-Aug-2013
21. Lufthansa-823212 from 17-Jul-1995 to 23-Aug-2013
22. Merck-659990 from 13-Nov-1995 to 23-Aug-2013
23. RWE-703712 from 17-Jul-1995 to 23-Aug-2013
24. SAP-716460 from 17-Jul-1995 to 23-Aug-2013
25. Siemens-723610 from 17-Jul-1995 to 23-Aug-2013
26. Thyssen-750000 from 17-Jul-1995 to 24-Feb-2014
27. Volkswagen-766403 from 17-Jul-1995 to 23-Aug-2013
28. DAX-846900 - from 03-Jun-1996 to 03-Sep-2013

C.0.2 NASDAQ

And daily close price of following stocks from NASDAQ index:

1. Avago-A0X9TN from 16-Sep-2009 to 14-Apr-2014
2. Baidu-A0F5DE from 08-Aug-2005 to 17-Apr-2014
3. Bed-Bath-884304 from 12-Sep-1997 to 16-Apr-2014
4. Biogen-789617 from 22-Jan-1998 to 17-Apr-2014
5. Broadcom-913684 from 22-Apr-1998 to 14-Apr-2014
6. Catamaran-A1J08W from 14-Apr-2009 to 04-Apr-2014
7. CA-Techn-A0JC59 from 14-Nov-1996 to 14-Apr-2014
8. Celgene-881244 from 10-Feb-2000 to 17-Apr-2014
9. Cerner-892807 from 06-Oct-1998 to 16-Apr-2014
10. Charter-Comm-A0YF1T from 20-Dec-2011 to 21-Feb-2014
11. Check-Point-901638 from 12-Jan-1998 to 16-Apr-2014
12. Cisco-878841 from 07-Feb-1997 to 17-Apr-2014

13. Citrix-898407 from 21-May-1997 to 17-Apr-2014
14. Cognizant-915272 from 02-Jul-1999 to 17-Apr-2014
15. Comcast-157484 from 21-Nov-2002 to 16-Apr-2014
16. Costco-888351 from 14-Jul-1997 to 10-Apr-2014
17. DirecTV-A1J1EZ from 14-Apr-2004 to 14-Apr-2014
18. Discovery-Comm-A0Q90G from 27-Jul-2005 to 07-Mar-2014
19. Dish-Network-A0NBN0 from 07-Jun-1999 to 29-Nov-2012
20. Doller-Tree-A0NFQC from 07-Feb-2000 to 11-Apr-2014
21. eBay-916529 from 29-Sep-1998 to 17-Apr-2014
22. Equinix-165241 from 21-Aug-2000 to 17-Apr-2014
23. Expedia-A1JRLJ from 19-Aug-2005 to 16-Apr-2014
24. Expeditors-875272 from 05-Aug-1999 to 30-Jan-2014
25. Express-Scripts-A1JWJL from 26-Jul-1999 to 17-Apr-2014
26. F5-922977 from 02-Jul-1999 to 11-Apr-2014
27. Facebook-A1JWVX from 18-May-2012 to 17-Apr-2014
28. Fastenal-887891 from 02-Mar-2001 to 15-Apr-2014
29. Fiserv-881793 from 13-Jul-1998 to 15-Apr-2014
30. Garmin-A1C06B from 21-Dec-2000 to 17-Apr-2014
31. Gilead-885823 from 16-Aug-1996 to 17-Apr-2014
32. Google-A0B7FY from 19-Aug-2004 to 17-Apr-2014
33. Henry-Schein-897961 from 25-Jan-2000 to 16-Apr-2014
34. Illumina-927079 from 02-Aug-2000 to 17-Apr-2014
35. Intel-855681 from 18-Nov-1996 to 17-Apr-2014
36. Intuit-886053 from 26-Sep-1997 to 17-Apr-2014
37. Intuitivr-Surgical-888024 from 24-Jul-2000 to 16-Apr-2014
38. Keurig-A1XFME from 22-Nov-2000 to 16-Apr-2014
39. KLA-Tencor-865884 from 05-Jan-1998 to 08-Apr-2014

40. Kraft-Foods-A1J20U from 04-Oct-2012 to 17-Apr-2014
41. Liberty-A0JMPL from 07-Nov-2006 to 07-Apr-2014
42. Liberty-Global-A1W0FL from 23-Dec-2004 to 11-Apr-2014
43. Liberty-Media-Corp-A1KBFW from 06-Mar-2013 to 14-Apr-2014
44. Linear-872629 from 26-Sep-1997 to 17-Apr-2014
45. Marriott-913070 from 20-Jul-1998 to 15-Apr-2014
46. Mattel-851704 from 29-Dec-1997 to 17-Apr-2014
47. Maxim-Integrated-876158 from 11-Jul-1996 to 05-Aug-2013
48. Micron-869020 from 27-Dec-1995 to 17-Apr-2014
49. Microsoft-870747 from 25-Sep-1996 to 17-Apr-2014
50. Mondelez-A1J4U0 from 19-Sep-2001 to 17-Apr-2014
51. Monster-Beverage-A1JSKK from 23-Feb-2004 to 17-Apr-2014
52. Mylan-868270 from 19-Jan-1998 to 16-Apr-2014
53. NetApp-A0NHKR from 11-Nov-1998 to 17-Apr-2014
54. NETFlix-552484 from 07-Oct-2002 to 17-Apr-2014
55. NVIDIA-918422 from 04-Feb-1999 to 17-Apr-2014
56. NXP-Semiconductors-A1C5WJ from 08-Sep-2010 to 17-Apr-2014
57. OReilly-A1H5JY from 20-Dec-2001 to 28-Mar-2014
58. PACCAR-861114 from 27-Dec-1999 to 11-Apr-2014
59. Paychex-868284 from 07-Jan-1998 to 15-Apr-2014
60. priceline-766054 from 06-Apr-1999 to 17-Apr-2014
61. Qualcomm-883121 from 17-Jul-1997 to 17-Apr-2014
62. Regeneron-Pharma-881535 from 28-Feb-2000 to 17-Apr-2014
63. Robinson-Worldwide-A0HGF5 from 16-May-2003 to 17-Apr-2014
64. Ross-870053 from 10-Jul-1998 to 16-Apr-2014
65. Sandisk-897826 from 19-Dec-1997 to 17-Apr-2014
66. SBA-Comm-923376 from 08-Jul-1999 to 17-Apr-2014

67. Seagate-A1C08F from 23-Jul-2003 to 16-Apr-2014
68. Sigma-Aldrich-863120 from 31-Jul-1998 to 15-Apr-2014
69. Staples-876951 from 26-Sep-1997 to 16-Apr-2014
70. Starbucks-884437 from 18-Dec-1997 to 17-Apr-2014
71. Stericycle-902518 from 30-Aug-2000 to 14-Apr-2014
72. Symantec-879358 from 04-Mar-1997 to 17-Apr-2014
73. Syrius-A1W8XE from 14-May-1998 to 17-Apr-2014
74. Tesla-A1CX3T from 01-Jul-2010 to 17-Apr-2014
75. Texas-Instruments-852654 from 05-Feb-1997 to 17-Apr-2014
76. Tractor-889826 from 17-Apr-2002 to 04-Feb-2014
77. TripAdvisor-A1JRLK from 06-Jan-2012 to 15-Apr-2014
78. Twenty-First-Century-A1WZP6 from 08-Feb-2002 to 17-Apr-2014
79. Verisk-A0YA2M from 04-Jan-2010 to 14-Feb-2014
80. Vertex-882807 from 04-Apr-2000 to 17-Apr-2014
81. Viacom-A0HM1Q from 05-Jan-1998 to 26-Mar-2014
82. VimpelCom-A0YE2R from 26-Apr-2010 to 17-Apr-2014
83. Vodafon-A1XD9Z from 01-Mar-2000 to 15-Apr-2014
84. Western-Digital-863060 from 23-Jan-1997 to 17-Apr-2014
85. Whole-Foods-886391 from 05-May-1998 to 17-Apr-2014
86. Wynn-Resorts-663244 from 05-May-2004 to 17-Apr-2014
87. Xilinx-880135 from 13-May-1996 to 07-Apr-2014
88. Yahoo-900103 from 02-May-1996 to 17-Apr-2014

Appendix D

Tests

This section contains list of all tests, it's components, BUY and SELL signal conditions.

D.1 Test 03 DAX

- profitUsingReturnsAndHurst
- DAX proove of hurst
- ResturnNDays 8
- Moving Hurst 64, 8
- BUY signal:
  `chaos_series(iChaos) > levelTrend AND returns(iReturn) > 0`
- SELL signal:
  `(chaos_series(iChaos) < levelVolatility)`

D.2 Test 04 NASDAQ

- profitUsingReturnsAndHurst
- NASDAQ proove of hurst
- ResturnNDays 8
- Moving Hurst 64, 8
- BUY signal:
  `chaos_series(iChaos) > levelTrend AND returns(iReturn) > 0`
- SELL signal:
  `chaos_series(iChaos) < levelVolatility`

D.3 Test 05 NASDAQ teach and test

- profitUsingReturnsAndHurst3
- NASDAQ teach and test 1. 1/2 teach 2. 1/2 test
- ResturnNDays 8
- Moving Hurst 64, 8
- BUY signal:
  `chaos_series(iChaos) > levelTrend AND returns(iReturn) > 0`
- SELL signal:
  `chaos_series(iChaos) < levelVolatility`

D.4 Test 06 DAX teach and test

- profitUsingReturnsAndHurst3
- DAX teach and test 1. 1/2 teach 2. 1/2 test
- ResturnNDays 8
- Moving Hurst 64, 8
- BUY signal:
  `chaos_series(iChaos) > levelTrend AND returns(iReturn) > 0`
- SELL signal:
  `chaos_series(iChaos) < levelVolatility`

D.5 Test 07 DAX teach and test

- profitUsingReturnsAndHurst4
- DAX teach and test 1. 1/2 teach 2. 1/2 test
- ResturnNDays 16
- Moving Hurst 64, 8
- BUY signal:
  `chaos_series(iChaos) > levelTrend AND returns(iReturn) > 0`
- SELL signal:
  `chaos_series(iChaos) < levelVolatility`

D.6 Test 07 DAX teach and test

- profitUsingReturnsAndHurst4
- DAX teach and test 1. 1/2 teach 2. 1/2 test
- ResturnNDays 32
- Moving Hurst 64, 8
- BUY signal:
  `chaos_series(iChaos) > levelTrend AND returns(iReturn) > 0`
- SELL signal:
  `chaos_series(iChaos) < levelVolatility`

D.7 Test 08 teach and test

- profitUsingReturnsAndHurst5
- DAX teach and test 1. 1/2 teach 2. 1/2 test
- Moving Average Convariance 64 days
- ResturnNDays 32
- Moving Hurst 128, 8
- BUY signal:
  `chaos_series(iChaos) > levelTrend AND returns(iReturn) > 0`
- SELL signal:
  `chaos_series(iChaos) < levelVolatility`

D.8 Test 09 NASDAQ teach and test

- profitUsingReturnsAndHurst5
- NASDAQ teach and test 1. 1/2 teach 2. 1/2 test
- Moving Average Convariance 64 days
- ResturnNDays 32
- Moving Hurst 128, 8
- BUY signal:
  `chaos_series(iChaos) > levelTrend AND returns(iReturn) > 0`
- SELL signal:
  `chaos\_series(iChaos) < levelVolatility`

D.9 Test 10 NASDAQ teach and test

- profitUsingReturnsAndHurst6
- NASDAQ teach and test 1. 1/2 teach 2. 1/2 test
- Moving Average Convariance 64 days
- ResturnNDays 32
- Moving Hurst 128, 8
- BUY signal:
  `chaos_series(iChaos) > levelTrend AND returns(iReturn) > 0`
- SELL signal:
  `chaos_series(iChaos) < levelVolatility`

D.10 Test 11 DAX teach and test

- profitUsingReturnsAndHurst6
- DAX teach and test 1. 1/2 teach 2. 1/2 test
- Moving Average Convariance 64 days
- ResturnNDays 32
- Moving Hurst 128, 8
- BUY signal:
  `chaos_series(iChaos) > levelTrend AND returns(iReturn) > 0`
- SELL signal:
  `chaos_series(iChaos) < levelVolatility`

D.11 Test 12 DAX teach and test

- profitUsingReturnsAndHurst7
- DAX teach and test 1. 1/2 teach 2. 1/2 test
- Moving Average Convariance 32 days
- ResturnNDays 8
- Moving Hurst 64, 8

- BUY signal:
  `chaos_series(iChaos) > levelTrend AND Close(iPrice) > avg_close(iAvg)`
- SELL signal:
  `chaos_series(iChaos) < levelVolatility)`
- SELL signal:
  `Close(iPrice) < avg_close(iAvg)`

D.12 Test 13 NASDAQ teach and test

- profitUsingReturnsAndHurst7
- DAX teach and test 1. 1/2 teach 2. 1/2 test
- Moving Average Convariance 32 days
- ResturnNDays 8
- Moving Hurst 64, 8
- BUY signal:
  `chaos_series(iChaos) > levelTrend AND Close(iPrice) > avg_close(iAvg)`
- SELL signal:
  `chaos_series(iChaos) < levelVolatility`
- SELL signal:
  `Close(iPrice) < avg_close(iAvg)`

D.13 Test 14 DAX teach and test

- profitUsingReturnsAndHurst8
- DAX teach and test 1. 1/2 teach 2. 1/2 test
- Moving Average Convariance 32 days
- ResturnNDays 8
- Moving Hurst 64, 8
- BUY signal:
  `Close(iPrice) > avg_close(iAvg)`
- SELL signal:
  `Close(iPrice) < avg_close(iAvg)`

D.14 Test 15 DAX teach and test

- profitUsingReturnsAndHurst9
- DAX teach and test 1. 100 days teach 100 days test
- Moving Average Convariance 32 days
- ResturnNDays 8
- Moving Hurst 64, 8
- Teach length 100
- BUY signal:
  `chaos_series(iChaos) > levelTrend AND returns(iReturn) > 0`
- SELL signal:
  `chaos_series(iChaos) < levelVolatility`

D.15 Test 16 DAX teach and test

- profitUsingReturnsAndHurst9
- DAX teach and test 1. 100 days teach 100 days test
- Moving Average Convariance 32 days
- ResturnNDays 8
- Moving Hurst 64, 8
- Teach length 100
- BUY signal:
  `chaos_series(iChaos) > levelTrend AND returns(iReturn) > 0`
- SELL signal:
  `chaos_series(iChaos) < levelVolatility`

D.16 Test 17 DAX MACD

- profitUsingMACD
- DAX
- BUY signal:
  `macdvec(iPrice) > nineperma(iPrice)`
- SELL signal:
  `macdvec(iPrice) <= nineperma(iPrice)`

D.17 Test 18 NASDAQ MACD

- profitUsingMACD
- NASDAQ
- BUY signal:
  `macdvec(iPrice)  > nineperma(iPrice)`
- SELL signal:
  `macdvec(iPrice) <= nineperma(iPrice)`

D.18 Test 19 DAX MHCD

- profitUsingReturnsAndHurst10
- DAX
- returnDays = 8
- hurstDays = 8 $\rightarrow$ 64
- hurstDays2 = 16 $\rightarrow$ 128
- BUY signal:
  `chaos_series_2(iChaos_2)  > chaos_series(iChaos) AND returns(iReturn) > 0`
- SELL signal:
  `chaos_series_2(iChaos_2) <= chaos_series(iChaos)`

D.19 Test 20 NASDAQ MHCD

- profitUsingReturnsAndHurst10
- NASDAQ
- returnDays = 8
- hurstDays = 8 $\rightarrow$ 64
- hurstDays2 = 16 $\rightarrow$ 128
- BUY signal:
  `chaos_series_2(iChaos_2) > chaos_series(iChaos) AND returns(iReturn > 0)`
- SELL signal:
  `chaos_series_2(iChaos_2) <= chaos_series(iChaos)`

D.20 Test 21 DAX MHCD

- profitUsingReturnsAndHurst11
- DAX
- smooth(Close)
- returnDays = 8
- hurstDays = 8 → 64
- hurstDays2 = 16 → 128
- BUY signal:
  `chaos_series_2(iChaos_2) > chaos_series(iChaos) AND returns(iReturn) > 0`
- SELL signal:
  `chaos_series_2(iChaos_2) <= chaos_series(iChaos)`

D.21 Test 22 NASDAQ MHCD

- profitUsingReturnsAndHurst11
- NASDAQ
- smooth(Close)
- returnDays = 8
- hurstDays = 8 → 64
- hurstDays2 = 16 → 128
- BUY signal:
  `chaos_series_2(iChaos_2) > chaos_series(iChaos) AND returns(iReturn) > 0`
- SELL signal:
  `chaos_series_2(iChaos_2) <= chaos_series(iChaos)`

D.22 Test 23 DAX MHCD

- profitUsingReturnsAndHurst12
- DAX
- smooth(Close)
- hurstDays = 8 → 64

- hurstDays2 = 16 → 128
- BUY signal:
  `chaos_series_2(iChaos_2) > chaos_series(iChaos)`
  `AND (CloseFiltered(iPrice) - closeFiltered(iPrice - 1)) > 0`
- SELL signal:
  `chaos_series_2(iChaos_2) <= chaos_series(iChaos)`

D.23 Test 24 NASDAQ MHCD

- profitUsingReturnsAndHurst12
- NASDAQ
- smooth(Close)
- hurstDays = 8 → 64
- hurstDays2 = 16 → 128
- BUY signal:
  `chaos_series_2(iChaos_2) > chaos_series(iChaos)`
  `AND (CloseFiltered(iPrice) - closeFiltered(iPrice - 1)) > 0`
- SELL signal:
  `chaos_series_2(iChaos_2) <= chaos_series(iChaos)`

D.24 Test 25 DAX MHCD

- profitUsingReturnsAndHurst13
- DAX
- returnDays = 1 → 5
- hurstDays = 16 → 96
- hurstDays2 = 32 → 192
- BUY signal:
  `chaos_series_2(iChaos_2) > chaos_series(iChaos)`
  `AND (Close(iPrice) - Close(iPrice - returnDays)) > 0`
- SELL signal:
  `chaos_series_2(iChaos_2) <= chaos_series(iChaos)`

D.25 Test 26 NASDAQ MHCD

- profitUsingReturnsAndHurst13
- NASDAQ
- returnDays = $1 \to 5$
- hurstDays = $16 \to 96$
- hurstDays2 = $32 \to 192$
- BUY signal:
  `chaos_series_2(iChaos_2) > chaos_series(iChaos)`
  `AND (Close(iPrice) - Close(iPrice - returnDays)) > 0`
- SELL signal:
  `chaos_series_2(iChaos_2) <= chaos_series(iChaos)`

D.26 Test 27 DAX MHCD

- profitUsingReturnsAndHurst14
- DAX
- smooth(Close)
- returnDays = $1 \to 10$
- hurstDays = $8 \to 64$
- hurstDays2 = $16 \to 128$
- BUY signal:
  `chaos_series_2(iChaos_2) > chaos_series(iChaos)`
  `AND (CloseFiltered(iPrice) - closeFiltered(iPrice - returnDays)) > 0`
- SELL signal:
  `chaos_series_2(iChaos_2) <= chaos_series(iChaos)`

D.27 Test 28 NASDAQ MHCD

- profitUsingReturnsAndHurst14
- NASDAQ
- smooth(Close)
- returnDays = $1 \to 10$

- hurstDays = 8 → 64
- hurstDays2 = 16 → 128
- BUY signal:
  `chaos_series_2(iChaos_2) > chaos_series(iChaos)`
  `AND (CloseFiltered(iPrice) - closeFiltered(iPrice - returnDays)) > 0`
- SELL signal:
  `chaos_series_2(iChaos_2) <= chaos_series(iChaos)`

D.28 Test 29 DAX MHCD

- profitUsingReturnsAndHurst15
- DAX
- smooth(Close)
- returnDays = 1 → 10
- hurstDays = 8 (RafalWeronHurst, `chaos_sample_size = 2`)
- hurstDays2 = 16 (RafalWeronHurst, `chaos_sample_size = 2`)
- BUY signal:
  `chaos_series_2(iChaos_2) > chaos_series(iChaos)`
  `AND (CloseFiltered(iPrice) - closeFiltered(iPrice - returnDays)) > 0`
- SELL signal:
  `chaos_series_2(iChaos_2) <= chaos_series(iChaos)`

D.29 Test 30 NASDAQ MHCD

- profitUsingReturnsAndHurst15
- NASDAQ
- smooth(Close)
- returnDays = 1 → 10
- hurstDays = 8 (RafalWeronHurst, `chaos_sample_size = 2`)
- hurstDays2 = 16 (RafalWeronHurst, `chaos_sample_size = 2`)
- BUY signal:
  `chaos_series_2(iChaos_2) > chaos_series(iChaos)`
  `AND (CloseFiltered(iPrice) - closeFiltered(iPrice - returnDays)) > 0`

- SELL signal:
  `chaos_series_2(iChaos_2) <= chaos_series(iChaos)`

D.30 Test 31 DAX MHCD

- profitUsingReturnsAndHurst16
- DAX
- smooth(Close)
- returnDays = 1
- hurstDays = 16
- hurstDays2 = 32
- BUY signal:
  `chaos_series_2(iChaos_2) > chaos_series(iChaos)`
  `AND (CloseFiltered(iPrice) - closeFiltered(iPrice - returnDays)) > 0`
- SELL signal:
  `chaos_series_2(iChaos_2) <= chaos_series(iChaos)`
- Profit with fee from investment 2000

D.31 Test 32 NASDAQ MHCD

- profitUsingReturnsAndHurst16
- NASDAQ
- smooth(Close)
- returnDays = 1
- hurstDays = 16
- hurstDays2 = 32
- BUY signal:
  `chaos_series_2(iChaos_2) > chaos_series(iChaos)`
  `AND (CloseFiltered(iPrice) - closeFiltered(iPrice - returnDays)) > 0`
- SELL signal:
  `chaos_series_2(iChaos_2) <= chaos_series(iChaos)`
- Profit with fee from investment 2000

D.32 Test 32 DAX MHCD

- profitUsingReturnsAndHurst17
- DAX
- smooth(Close)
- returnDays = 1
- hurstDays = 16
- hurstDays2 = 32
- BUY signal:
  `chaos_series_2(iChaos_2) <= chaos_series(iChaos)`
  `AND (CloseFiltered(iPrice) - closeFiltered(iPrice - returnDays)) > 0`
- SELL signal:
  `chaos_series_2(iChaos_2) > chaos_series(iChaos)`
- Profit with fee from investment 2000

D.33 Test 33 NASDAQ MHCD

- profitUsingReturnsAndHurst17
- NASDAQ
- smooth(Close)
- returnDays = 1
- hurstDays = 16
- hurstDays2 = 32
- BUY signal:
  `chaos_series_2(iChaos_2) <= chaos_series(iChaos)`
  `AND (CloseFiltered(iPrice) - closeFiltered(iPrice - returnDays)) > 0`
- SELL signal:
  `chaos_series_2(iChaos_2) > chaos_series(iChaos)`
- Profit with fee from investment 2000